

CONTRIBUTIONS FROM THE OBSERVATORY OF
COLUMBIA UNIVERSITY, NEW YORK.—No. 15.

JOHN K. REES, Director.

THE PRÆSEPE GROUP;
MEASUREMENT AND REDUCTION
OF THE
RUTHERFURD PHOTOGRAPHS.

BY
FRANK SCHLESINGER.

Submitted in partial fulfilment of the requirements for the degree of Doctor
of Philosophy in the Faculty of Pure Science, Columbia University.

NEW YORK:
1898.

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PREFACE.

The late Dr. B. A. Gould presented to the National Academy of Sciences, April 14, 1870, a paper on the "Reduction of Photographic Observations of the Præsepe." In this memoir Dr. Gould gave his discussion of six plates taken by Rutherford on three nights in February and April, 1867. The plates were made with the 11 $\frac{1}{4}$ -inch object glass and were measured under Rutherford's direction with his own measuring machine. The eight plates discussed in Mr. Schlesinger's paper were taken in 1870 and 1877 with the 13-inch object glass by Rutherford. These plates remained unmeasured until the present time, when the measurements were made with the Repsold measuring machine by Mr. Schlesinger with the help of Walter C. Kretz, A.M., and William H. Hays, A.M. The observatory is under obligations to these gentlemen for their interested and gratuitous services. The funds contributed by Rutherford Stuyvesant, Esq., have aided in the publication of this paper by the New York Academy of Sciences.

JOHN K. REES,
Professor of Astronomy
and Director of the Observatory,
Columbia University.

NEW YORK CITY, May 10, 1898.

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VI.—*The Præsepe Group; Measurement and Reduction of the Rutherford Photographs.*

BY FRANK SCHLESINGER.

Read April 4, 1898.

I.

Description of the Plates.

The collection of astronomical photographs presented by the late Lewis M. Rutherford to the Observatory of Columbia University contains eleven photographs of Præsepe taken with his larger and improved instrument; only eight of these were measured and reduced, three having been judged inferior to the rest. According to Rutherford's invariable practice, each plate shows two complete pictures of the group separated by about a millimetre in right ascension, the driving clock of the instrument having been stopped for a few seconds after the completion of the first or eastern impression. Near the west edge of the plate still a third image of each of the brighter stars in the group is found, separated by about forty millimetres from the two other impressions, the driving clock having been stopped for an interval of about three minutes after the completion of the second impression, and then started again and allowed to run long enough to permit the brighter stars to leave well-defined images. The object in securing these third impressions or *trails* was to afford means for orienting the group, but in the present work they were not used for this purpose, the orientation having been effected in another way. It is important, however, to know how accurately the use of trails will give the orientation, and they were therefore completely measured and reduced, and the results compared with those obtained by the method actually employed, which is of unquestioned accuracy but may not be always available.

A perpendicular to the plate passing through the optical centre of the object glass pierces the plate at a point whose approximate position must be known in order to reduce the measures of the stars to right ascensions and declinations. Rutherford so adjusted his plate holder that this point coincides approximately with the image of the central star of the group, numbered 15 in the following pages.

Table I gives the data of exposure for the plates. Plates VI, X and XI do not appear, these being the ones that were not measured, on account of their inferiority. This is due to the fact that the photographic images of the stars on these plates, when viewed under the microscope of the measuring machine were neither so round nor so well defined as on the other plates and therefore did not admit of so accurate measurement. The irregularity of the images is not due to a deterioration of the plates since they were taken, but to the bad behavior of RUTHERFURD's clock during the exposures. For this reason these three plates were never measured, it being deemed probable that more reliable results are to be obtained from the eight plates actually reduced, than if all the plates had been measured, in spite of the greater number of observations in the latter case. In this connection I should also say that not all the stars which appear on the plates were measured. A few whose images come near the edges of the plates were rejected, for not only are these images much distorted, but as we shall see later, the corrections become uncertain as we recede from the centre of the plate.

TABLE I.—PHOTOGRAPHS OF PRÆSEPE.

Observatory of L. M. Rutherford, New York.

Lat. = $40^{\circ}43'48''.5$ Long. = $4^{\text{h}}55^{\text{m}}56^{\text{s}}.62$ W.

No.	Date.	Sidereal Time.	Bar.	Thermometers.			Focus
				Att.	Ext.	Tel.	
		h m s					
I	1870 Apr. 24	10 45 05	30.01	60°	55°	58°	8.4
II	1870 Apr. 24	11 25 35	30.01	60	55	58	8.4
III	1870 Apr. 25	11 10 35	30.26	53	47	53	8.4
IV	1870 Apr. 25	11 59 35	30.26	53	47	53	8.4
V	1877 Apr. 14	10 39 38	30.06	47	45	48	7.8
VII	1877 Apr. 25	11 26 02	30.06	57	56	58	7.7
VIII	1877 Apr. 25	11 53 32	30.06	57	56	58	7.7
IX	1877 May 2	10 57 08	29.86	47	46	48	7.8

The column marked "sidereal time" gives the mean of four instants for each plate: beginning of east exposure, end of east exposure, beginning of west exposure and end of west exposure; each exposure lasted six minutes. Three thermometers were read: attached, external and telescope, the last being in contact

with the telescope-tube. The last column, marked "focus," gives the reading of a micrometer head attached to the eye end of the telescope and shows the position of the plate holder; this information is not used in the reductions and is given only to provide for the possibility of determining a relation between this reading and the scale-value after a sufficient number of the photographs made with this instrument has been reduced.

II.

Measurement of the Plates.

The plates were measured with the older Repsold Measuring Machine of this Observatory, which is a counterpart of the one by the same maker belonging to the University of Leyden except that an alteration has been made which obviates "projection errors." (See III.) A full description of the Leyden machine is given in the "Bulletin du Comité Permanent," Vol. 1, page 169, and also in the recent work by Dr. Scheiner, "Photographie der Gestirne," page 148. The machine is so constructed that the position of a star may be determined either by position angle and distance or by rectangular coördinates; the latter method was adopted in the present case. A star which is to be measured may be brought into the field of the reading microscope by moving the plate along a straight guiding cylinder and then moving the microscope at right angles to the cylinder on another straight guiding way. The wire of the micrometer is made to bisect the image of the star and the micrometer head is read. Then the whole microscope is revolved through a small vertical angle and the wire set upon a scale of millimetres placed parallel to the motion of the microscope. The difference of the two readings on star and scale, together with the number of the line on the scale gives us the position of the star. Having gone through the same operation for all the stars we obtain their relative positions, at least in one direction; the plate is now revolved through 90° by means of the graduated circle and the stars are again measured; these two sets of measures are sufficient to fix the relative positions of the stars, but in order to secure greater accuracy and especially to eliminate personality the plate is turned 180° and 270° respectively from its original position, and the stars are read a third and a fourth time. By means of the trails

or otherwise the plate may be so placed in the machine that a circle of declination through the central star shall be approximately parallel to the guiding cylinder; in this way we obtain rectangular coördinates which are nearly in the directions of right ascensions and declinations, thus rendering their conversion into the latter a comparatively easy matter.

Two observers alternated in the measurements, one recording while the other observed; the details of each morning's work, which usually lasted a little over two hours, are as follows: the first observer reads the circle, runs and temperature, the second reads on the central star thus: East image, scale, scale, east image; west image, scale, scale, west image. Continuing, the second observer goes through the same operations for usually three other stars, experience having shown that four stars could be read conveniently without fatiguing the eye; thus the observers alternate till twenty or twenty-five stars have been read and then the temperature is recorded a second time. The morning's work is now half finished; the same stars are then observed in the reverse order, care being taken that each observer shall now read those stars that he had not read in the first half; having finally gotten back to the central star, temperature, runs and the circle are read as at the beginning. This process of repeating all the measurements in the reverse order, eliminates the effects of any change in the machine or in the observers that is proportionate to the time, for the mean of the two times of observation is nearly the same for all the stars. In the first half of the morning's work the micrometer head is set at about $9.^{R}0$ when pointed at a star; but in the second half the reading is made $9.^{R}5$; in this way periodic errors of the screw are nearly eliminated, for both star and scale are read with two different parts of the screw separated by half a turn.

The measurements made in the first position of the plate, *i. e.*, with the stars having the greatest *right ascensions* farthest from the cylinder are recorded as "*x* direct;" on the next day the plate is turned 90° in a counter-clockwise direction so that now the stars having the greatest *north polar distances* are farthest from the cylinder. The measurements taken in this position are called "*y* direct," while those taken in the two opposite positions, 180° and 270° from the original position are called respectively "*x* reversed" and "*y* reversed." As only twenty or twenty-five stars could be

measured on each day, and as the photographs of Præsepe show about forty-five stars that admit of measurement, it was necessary to spend two days on each position of the plate. To eliminate the effect of a possible motion of the plate or of the scale the central star 15 was read each day by both observers. After three of the plates had been measured, viz. : III, VII and IX, it was decided to read the central star more often, so that on the succeeding plates four such readings were made every day, instead of two.

Three observers were engaged in the measurement of the first five plates, but only two were concerned in the work for any single day. Care was always taken to have the same pair of observers make both the direct and the reversed measurements of a particular set of stars, in order to eliminate personality. Suppose one of the observers has contracted the habit of always setting the micrometer wire too far to the right of the centre of a star's image by an amount depending upon the size of the image; the distance between two stars as obtained by such an observer will be subject to an error which depends upon the difference of magnitudes of the stars. But when the plate is reversed 180° , the same observer will get a distance which is too small by as much as the first distance was too large or vice versa; consequently the mean of the two measurements will be free from such personality as we have supposed. However, this method of measurement does not eliminate all personality, for the star images are seldom round and are usually more sharply defined on one edge than upon the other; two observers will thus sometimes differ considerably in their estimations of the true centre of the image.

Table II gives the runs, circle reading, etc., for each day. Runs were observed twice daily, once before and once after the measurement of the stars; the number in the column headed "runs" is the mean of the two determinations expressed in millimetres. The circle was also read twice, employing two microscopes 180° apart for each reading; in the column marked "circle" the degrees and minutes are always taken from the right-hand microscope, while the number of seconds is the mean of both microscopes. The thermometer occupied a fixed position near the plate and was graduated in Fahrenheit degrees. The last column gives the initials of the three observers, Kretz, Hays and Schlesinger.

TABLE II.—DAILY RECORDS.

Date, 1897.	Runs in mm.	Circle.	Ther.	Position of Plate, and Stars Measured.	Obs'rs.
Plate III.					
Jan. 12	—0.0029	185° 58' 00"	63.3	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-24, 26, 28, 29, 45.	S, K
" 13	—0.0026	275 57 57	67.0	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-26, 28, 29, 33.	S, H
" 18	—0.0026	5 58 00	63.2	<i>x</i> reversed; see Jan. 12.	S, K
" 19	—0.0021	185 57 57	60.8	<i>x</i> direct; 1, 2, 5, 7, 15, 23A, 25, 27, 31-37; 39, 40, 43-45.	K, H
" 20	—0.0016	95 58 02	63.6	<i>y</i> reversed; see Jan. 13.	S, H
" 21	—0.0025	275 58 01	65.2	<i>y</i> direct; 1, 5, 7, 15, 23A, 27, 31-37, 39, 40, 43-45.	S, H
" 23	—0.0035	5 58 00	60.5	<i>x</i> reversed; see Jan. 19.	K, H
" 26		275 58 00		Trails; 15, 23, 27, 31.	S, K
" 27	—0.0020	95 58 00	61.6	<i>y</i> reversed; see Jan. 21.	S, H
Plate IX.					
Feb. 5	+0.0028	177° 37' 02"	63.2	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 45.	S, K
" 8		267 37 01		Trails; 15, 23, 31, 37.	S, K
" 9	+0.0016	267 37 00	62.2	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 33.	K, H
" 10	+0.0010	267 36 58	63.6	<i>y</i> direct; 1, 5, 7, 15, 23A, 31-40, 43-45.	S, H
" 11	+0.0021	87 37 02	60.9	<i>y</i> reversed; see Feb. 10.	S, H
" 13	+0.0006	87 37 01	63.5	<i>y</i> reversed; see Feb. 9.	K, H
" 16	+0.0031	357 37 02	64.2	<i>x</i> reversed; see Feb. 19.	K, H
" 17	—0.0016	357 37 00	65.9	<i>x</i> reversed; see Feb. 5.	S, K
" 19	+0.0036	177 37 02	66.4	<i>x</i> direct; 1, 2, 5, 7, 15, 23A, 31-40, 43-45.	K, H
Plate VII.					
Feb. 26	+0.0026	86° 54' 00"	63.8	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 31-33, 45.	S, H
" 27	+0.0030	356 54 00	62.1	<i>y</i> reversed; see Mar. 1.	S, K
Mar. 1	+0.0039	176 54 02	59.4	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 31-33.	S, K
" 2	+0.0011	176 54 02	65.1	<i>y</i> direct; 1, 5, 7, 7A, 15, 19, 21, 23A, 33-45.	K, H
" 3	+0.0039	86 53 58	65.9	<i>x</i> direct; 1, 2, 5, 7, 7A, 15, 19, 21, 23A, 34-45.	S, H
" 4	+0.0044	266 53 56	66.8	<i>x</i> reversed; see Feb. 26.	S, H
" 5		176 54 00		Trails; 15, 22, 23, 31.	S, H
" 6	+0.0045	356 54 03	65.8	<i>y</i> reversed; see Mar. 2.	K, H
" 8	+0.0026	266 54 00	60.8	<i>x</i> reversed; see Mar. 3.	S, H

TABLE II.—DAILY RECORDS. (Continued.)

Date, 1897.	Runs in mm.	Circle.	Ther.	Position of Plate, and Stars Measured.	Obsr's.
Plate VIII.					
Mar. 15	+0.0036	177° 02' 28"	56.8	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 31, 45.	S, K
" 16	+0.0036	267 02 30	62.6	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22, 29, 31, 33.	K, H
" 17	+0.0039	357 02 31	60.4	<i>x</i> reversed; see Mar. 15.	S, K
" 18	+0.0036	357 02 27	62.3	<i>x</i> reversed; see Mar. 25.	S, H
" 20	+0.0050	87 02 32	62.0	<i>y</i> reversed; see Mar. 16.	K, H
" 22	+0.0045	87 02 28	64.6	<i>y</i> reversed; see Mar. 24.	S, K
" 23		267 02 29		Trails; 15, 22, 23, 31.	K, H
" 24	+0.0042	267 02 34	63.9	<i>y</i> direct; 1, 5, 7, 7A, 15, 19, 23A, 32-45.	S, K
" 25	+0.0045	177 02 29	62.8	<i>x</i> direct; 1, 2, 5, 7, 7A, 15, 19, 23A, 32-45.	S, H
Plate II.					
Apr. 3	+0.0015	276° 00' 00"	65.9	<i>y</i> direct; 2, 4, 6, 8, 10, 11, 14-18, 20, 22-26, 28, 29, 33.	K, H
" 5	+0.0015	185 59 59	65.9	<i>x</i> direct; 2, 4, 6, 8, 10, 11, 14-18, 20, 22-29, 45.	S, K
" 6	+0.0001	96 00 02	67.8	<i>y</i> reversed; see Apr. 3.	K, H
" 7	+0.0018	6 00 00	67.8	<i>x</i> reversed; see Apr. 5.	S, K
" 8	+0.0035	6 00 00	67.6	<i>x</i> reversed; see Apr. 10.	S, H
" 9	+0.0046	276 00 00	64.7	<i>y</i> direct; 1, 5, 7, 7A, 15, 19, 23A, 27, 31-40, 43-45.	K, H
" 10	+0.0055	185 59 58	65.6	<i>x</i> direct; 1, 2, 5, 7, 7A, 15, 19, 23A, 31-40, 43-45.	S, H
" 14	+0.0062	95 59 59	65.9	<i>y</i> reversed; see Apr. 9.	K, H
Plate IV.					
Apr. 24	+0.0101	186° 16' 58"	69.7	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 45.	S, K
" 28	+0.0109	6 17 01	65.9	<i>x</i> reversed; see Apr. 24.	S, K
" 29	+0.0151	276 17 00	66.9	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-26, 28, 29, 33.	S, K
May 1	+0.0165	96 17 02	67.5	<i>y</i> reversed; see Apr. 29.	S, K
" 3	+0.0155	96 16 58	65.0	<i>y</i> reversed; see May 5.	S, K
" 4	+0.0136	186 16 58	66.0	<i>x</i> direct; 1, 2, 5, 7, 7A, 15, 19, 23A, 31-40, 43-45.	S, K
" 5	+0.0156	276 16 57	66.9	<i>y</i> direct; 1, 5, 7, 7A, 15, 19, 23A, 27, 31-40, 43-45.	S, K
" 6	+0.0160	6 17 00	68.8	<i>x</i> reversed; see May 4.	S, K
" 8		276 17 01		Trails; 15, 23, 31, 37.	S, K

TABLE II.—DAILY RECORDS. (*Concluded.*)

Date, 1897.	Runs in mm.	Circle.	Ther.	Position of Plate, and Stars Measured.	Obs'rs.
Plate I.					
Nov. 3	—0.0080	187° 10' 05"	65.3	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29.	S, K
" 4	—0.0062	187 10 04	63.8	<i>x</i> direct; 1, 2, 5, 7, 15, 23A, 31-40, 43, 44.	S, K
" 6	—0.0076	277 10 06	65.4	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 33.	S, K
" 10	—0.0056	277 10 03	65.0	<i>y</i> direct; 1, 5, 7, 15, 23A, 31-40, 43, 44.	S, K
" 11	—0.0064	7 10 07	67.0	<i>x</i> reversed; see Nov. 3.	S, K
" 13	—0.0066	7 10 05	64.1	<i>x</i> reversed; see Nov. 4.	S, K
" 16	—0.0069	97 10 05	65.2	<i>y</i> reversed; see Nov. 6.	S, K
" 17		277 10 08		Trails; 15, 23, 31, 37.	S, K
" 18	—0.0065	97 10 05	64.6	<i>y</i> reversed; see Nov. 10.	S, K
Plate V.					
Nov. 20	—0.0041	176° 09' 07"	64.5	<i>x</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29, 45.	S, K
" 23	—0.0060	176 09 07	64.2	<i>x</i> direct; 1, 2, 5, 7, 15, 19, 23A, 31, 32, 34, 35, 37, 39, 40, 43-45.	S, K
" 24	—0.0061	266 09 07	68.7	<i>y</i> direct; 2-4, 6, 8, 10, 11, 14-18, 20, 22-29.	S, K
" 26	—0.0059	266 09 06	71.0	<i>y</i> direct; 1, 5, 7, 15, 19, 23A, 31, 32, 34, 35, 37, 39, 40, 43-45.	S, K
" 27	—0.0068	356 09 07	68.5	<i>x</i> reversed; see Nov. 20.	S, K
" 30	—0.0072	356 09 04	63.4	<i>x</i> reversed; see Nov. 23.	S, K
Dec. 1	—0.0065	86 09 07	66.1	<i>y</i> reversed; see Nov. 24.	S, K
" 2	—0.0058	86 09 08	65.4	<i>y</i> reversed; see Nov. 26.	S, K

On the next page is given a specimen of the recording sheets; all the measurements relating to the same star for any one plate are recorded on one sheet. The forms are not designed for the separate reduction of the two impressions of a group which appear on each plate, but the mean of the measurements on the two images of a star is taken at once; that is, each star is treated as though it occupied the middle point between its two images. The place of this point is known when we have given the lines of the scale used in the measurement and the quantity $\frac{1}{2} m$, which is obtained thus: from the reading on the scale we subtract the reading on the star, and the mean of these differences for both images is taken, giving "Mean of Diff's;" as the scale is one of millimetres and as *two* complete turns of the screw correspond to a space on the scale we divide by 2 and get $\frac{1}{2} m$. In reading on the scale it was made a rule to select the line having the next lower number; consequently the place of a star is obtained by *adding* $\frac{1}{2} m$ to the mean of the numbers of the lines used; in rare cases, however, the next higher line was employed when it came nearly opposite to a star, as on March 24; this has been indicated by affixing a minus sign to $\frac{1}{2} m$. In the *y* measurements the same line was always used for both images, since the latter differ only in apparent right ascension.

Table III gives the uncorrected observations; it is only necessary to set down the numbers of the lines used and $\frac{1}{2} m$, for these not only fix the place of the star but, as we shall see later, they are sufficient for the application of all instrumental corrections. The numbers of the stars are in the order of increasing right ascensions and are those given by Professor Schur,* excepting 7A and 23A, which do not appear in his triangulation of the group. The table gives the observations of $\frac{1}{2} m$ by both observers, followed by the difference reduced to seconds of arc in order to facilitate a comparison.

* "Astronomische Mittheilungen der K. Sternwarte zu Göttingen," part IV.

PRESEPE. PLATE VIII. STAR 7.

March 25, 1897; z direct.				March 24, 1897; y direct.				March 18, 1897; z reversed.				March 22, 1897; y reversed.			
Star.		Scale.		Star.		Scale.		Star.		Scale.		Star.		Scale.	
Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.
9.053	74	10.680	Diff's of	9.030	36	9.009	Diff's of	9.024	44	10.513	Diff's of	9.081	83	10.214	Diff's of
.050		.681	Mean	.030		.010	Mean	.035		.512	Mean	.085		.212	Mean
Mean 9.0515		10.6805	1.6290	9.0300		9.0095	-0.0205	9.0295		10.5125	1.4830	9.0830		10.2130	1.1300
West 9.050	76	9.280		9.050	36	9.009		9.013	43	9.910		9.070	83	10.212	
.051		.280		.050		.009		.023		.910		.069		.211	
Mean 9.0505		9.2800	0.2295	9.0500		9.0090	-0.0410	9.0180		9.9100	0.8920	9.0695		10.2115	1.1420
Mean of Diff's = 0.9292 $\frac{1}{2} m = 0.4646$				Mean of Diff's = -0.0368 $\frac{1}{2} m = -0.0184$				Mean of Diff's = 1.1875 $\frac{1}{2} m = 0.5938$				Mean of Diff's = 1.1360 $\frac{1}{2} m = 0.5680$			
Measured by Hays. Recorded by Schl.				Measured by Schl. Recorded by Kretz.				Measured by Schl. Recorded by Hays.				Measured by Schl. Recorded by Kretz.			
Star.		Scale.		Star.		Scale.		Star.		Scale.		Star.		Scale.	
Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.	Microm.	Line.	Line.	Microm.
9.597	74	11.229	Diff's of	9.547	36	9.515	Diff's of	9.550	44	11.021	Diff's of	9.555	83	10.681	Diff's of
.598		.230	Mean	.540		.518	Mean	.543		.020	Mean	.555		.684	Mean
Mean 9.5975		11.2295	1.6320	9.5435		9.5165	-0.0270	9.5465		11.0205	1.4740	9.5550		10.6825	1.1275
West 9.579	76	9.800		9.562	36	9.517		9.535	43	10.420		9.545	83	10.683	
.574		.800		.560		.517		.535		.419		.540		.683	
Mean 9.5765		9.8000	0.2235	9.5610		9.5170	-0.0440	9.5350		10.4195	0.8845	9.5425		10.6830	1.1405
Mean of Diff's = 0.9278 $\frac{1}{2} m = 0.4639$				Mean of Diff's = -0.0355 $\frac{1}{2} m = -0.0178$				Mean of Diff's = 1.1792 $\frac{1}{2} m = 0.5896$				Mean of Diff's = 1.1340 $\frac{1}{2} m = 0.5670$			
Measured by Schl. Recorded by Hays.				Measured by Kretz. Recorded by Schl.				Measured by Hays. Recorded by Schl.				Measured by Schl. Recorded by Kretz.			

TABLE III.—PLATE I: x MEASUREMENTS.

Star.	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	60,61	0.9016	0.8984	+0.19	57,58	0.6695	0.6681	+0.07
15	60,61	0.9010	0.8994	+ .07	57,58	0.6685	0.6696	— .06
2	94,95	0.4114	0.4114	.00	24,25	0.1652	0.1635	+ .08
3	93,94	0.2961	0.2954	+ .04	25,26	0.2798	0.2782	+ .08
4	91,92	0.5395	0.5419	— .13	27,28	0.0356	0.0348	+ .04
6	78,79	0.3274	0.3290	— .08	40,41	0.2472	0.2455	+ .08
8	71,72	0.4849	0.4878	— .15	47,48	0.0884	0.0849	+ .19
10	70,71	0.5885	0.5859	+ .14	48,49	— .0174	— .0169	— .03
11	69,70	0.7241	0.7252	— .06	48,49	0.8429	0.8451	— .12
14	61,62	0.1022	0.1020	+ .01	57,58	0.4649	0.4684	— .19
16	60	0.4836	0.4858	— .12	58,59	0.5859	0.5871	— .06
17	58,59	0.5726	0.5739	— .07	60,61	— .0020	— .0006	— .07
18	57,58	0.0299	0.0321	— .12	61,62	0.5404	0.5391	+ .07
20	56,57	0.6860	0.6865	— .03	61,62	0.8866	0.8865	.00
22	54,55	0.7451	0.7494	— .23	63,64	0.8208	0.8221	— .08
23	53,54	0.3750	0.3759	— .05	65,66	0.1946	0.1944	+ .01
24	51,52	0.2446	0.2451	— .03	67,68	0.3272	0.3256	+ .08
25	50,51	0.6644	0.6640	+ .02	67,68	0.9106	0.9080	+ .13
26	50,51	0.5456	0.5480	— .13	68,69	0.0282	0.0231	+ .27
27	50,51	0.2915	0.2942	— .14	68,69	0.2752	0.2784	— .17
28	50,51	0.0596	0.0622	— .14	68,69	0.5080	0.5075	+ .03
29	49,50	0.4759	0.4772	— .07	69,70	0.0936	0.0944	— .04
15	60,61	0.9005	0.8999	+0.02	57,58	0.6698	0.6682	+0.08
15	60,61	0.9009	0.8990	+ .12	57,58	0.6688	0.6669	+ .10
1	95,96	0.4069	0.4066	+ .02	23,24	0.1675	0.1684	— .05
2	94,95	0.4126	0.4120	+ .03	24,25	0.1625	0.1616	+ .05
5	82,83	0.5152	0.5181	— .15	36,37	0.0534	0.0540	— .03
7	75,76	0.5840	0.5830	+ .05	43,44	— .0130	— .0164	+ .18
23A	52,53	0.3599	0.3531	+ .36	66,67	0.2135	0.2155	— .11
31	48,49	0.8738	0.8725	+ .08	69,70	0.6971	0.6972	— .01
32	47,48	— .0078	— .0088	+ .05	71,72	0.5745	0.5799	— .29
33	45,46	0.3332	0.3376	— .23	73,74	0.2350	0.2410	— .32
34	44,45	0.6959	0.7006	— .25	73,74	0.8731	0.8708	+ .13
35	42,43	0.7008	0.7029	— .12	75,76	0.8659	0.8645	+ .08
36	41,42	0.2239	0.2221	+ .09	77,78	0.3489	0.3515	— .14
37	41,42	0.0792	0.0799	— .04	77,78	0.4944	0.4919	+ .13
38	37,38	0.4668	0.4691	— .12	81,82	0.1081	0.1061	+ .10
39	36,37	0.7224	0.7174	+ .26	81,82	0.8555	0.8554	.00
40	34,35	0.8836	0.8819	+ .08	83,84	0.6880	0.6890	— .05
43	27,28	— .0094	— .0124	+ .16	91,92	0.5789	0.5781	+ .04
44	26,27	0.4574	0.4624	— .26	92,93	0.1164	0.1168	— .02

TABLE III.—PLATE I: γ MEASUREMENTS.

Star.	γ direct.				γ reversed.			
	Line.	$\frac{1}{2}$ m. or Scale minus Star.		$S-K$	Line.	$\frac{1}{2}$ m. or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	52	0.3166	0.3169	—0.02	67	0.2538	0.2550	—0.06
15	52	0.3140	0.3176	— .19	67	0.2551	0.2545	+ .03
2	48	0.8969	0.8979	— .06	70	0.6722	0.6735	— .07
3	34	0.3140	0.3131	+ .05	85	0.2620	0.2640	— .11
4	17	0.2278	0.2255	+ .12	102	0.3502	0.3512	— .05
6	66	0.5180	0.5181	— .01	53	0.0520	0.0530	— .05
8	67	0.2651	0.2636	+ .08	52	0.3064	0.3069	— .03
10	45	0.2985	0.2952	+ .17	74	0.2756	0.2744	+ .06
11	39	0.5589	0.5594	— .03	80	0.0149	0.0134	+ .08
14	73	0.2576	0.2595	— .10	46	0.3138	0.3102	+ .19
16	18	0.0766	0.0752	+ .07	101	0.4996	0.5044	— .25
17	36	0.1639	0.1634	+ .03	83	0.4050	0.4050	.00
18	37	0.0128	0.0138	— .05	82	0.5598	0.5579	+ .10
20	36	0.2369	0.2336	+ .17	83	0.3358	0.3385	— .14
22	68	— .0124	— .0101	— .12	51	0.5811	0.5822	— .06
23	65	0.5135	0.5144	— .05	54	0.0558	0.0571	— .07
24	35	0.6510	0.6511	— .01	83	0.9205	0.9208	— .03
25	46	0.1918	0.1918	.00	73	0.3802	0.3779	+ .12
26	23	0.2064	0.2035	+ .15	96	0.3684	0.3689	— .03
27	45	0.0530	0.0535	— .03	74	0.5215	0.5174	+ .22
28	56	0.2920	0.2919	+ .01	63	0.2778	0.2786	— .04
29	79	0.9150	0.9150	.00	39	0.6569	0.6605	— .19
33	15	0.0512	0.0539	— .14	104	0.5241	0.5222	+ .10
15	52	0.3161	0.3172	—0.06	67	0.2540	0.2525	+0.08
15	52	0.3174	0.3172	+ .01	67	0.2535	0.2536	— .01
1	68	0.0005	0.0006	— .01	51	0.5711	0.5720	— .05
5	81	0.2686	0.2674	+ .06	38	0.3002	0.2992	+ .05
7	33	0.2762	0.2805	— .23	86	0.2989	0.2975	+ .07
23A	67	0.1061	0.1066	— .03	52	0.4675	0.4684	— .05
31	36	0.5514	0.5510	+ .02	83	0.0161	0.0155	+ .03
32	12	0.7218	0.7219	— .01	106	0.8538	0.8489	+ .26
33	15	0.0512	0.0515	— .02	104	0.5244	0.5242	+ .01
34	48	0.4732	0.4788	— .30	71	0.0906	0.0889	+ .09
35	85	0.7528	0.7519	+ .05	33	0.8198	0.8231	— .16
36	63	0.1924	0.1936	— .06	56	0.3759	0.3762	— .02
37	39	0.0658	0.0655	+ .02	80	0.5042	0.5071	— .15
38	56	0.0759	0.0802	— .23	63	0.4912	0.4900	+ .06
39	62	0.3799	0.3780	+ .10	57	0.1916	0.1932	— .08
40	17	0.4095	0.4081	+ .07	102	0.1689	0.1674	+ .08
43	59	0.2589	0.2636	— .25	60	0.3096	0.3105	— .05
44	78	0.6826	0.6850	— .13	40	0.8895	0.8870	+ .12

TABLE III.—(Continued.) PLATE II: x MEASUREMENTS.

Star.	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	63,64	0.7105	0.7096	+0.05	54,55	0.8424	0.8400	+0.12
15	63,64	0.7085	0.7099	— .07	54,55	0.8422	0.8426	— .02
2	97,98	0.2185	0.2192	— .04	21,22	0.3390	0.3380	+ .05
4	94,95	0.3538	0.3585	— .25	24,25	0.2039	0.1984	+ .29
6	81,82	0.1314	0.1352	— .20	37,38	0.4219	0.4200	+ .10
8	74,75	0.2901	0.2916	— .08	44,45	0.2625	0.2612	+ .07
10	73,74	0.3949	0.3975	— .14	45,46	0.1579	0.1584	— .03
11	72,73	0.5386	0.5364	+ .12	46	0.5189	0.5141	+ .25
14	64	0.4098	0.4100	— .01	54,55	0.6420	0.6401	+ .10
16	62,63	0.7976	0.7986	— .05	55,56	0.7551	0.7531	+ .10
17	61,62	0.3892	0.3901	— .05	57,58	0.1654	0.1615	+ .21
18	59,60	0.8448	0.8468	— .12	58,59	0.7078	0.7072	+ .03
20	59,60	0.5010	0.4991	+ .10	59,60	0.0539	0.0565	— .14
22	57,58	0.5558	0.5589	— .16	61	0.4941	0.4908	+ .19
23	56,57	0.1874	0.1871	+ .02	62,63	0.3664	0.3642	+ .12
24	54,55	0.0610	0.0612	— .01	64,65	0.4884	0.4902	— .10
25	53,54	0.4784	0.4780	+ .02	65,66	0.0745	0.0722	+ .12
26	53,54	0.3661	0.3669	— .04	65,66	0.1869	0.1885	— .09
27	53,54	0.1111	0.1118	— .04	65,66	0.4440	0.4401	+ .21
28	52,53	0.8736	0.8748	— .06	65,66	0.6789	0.6774	+ .08
29	52,53	0.2845	0.2878	— .17	66,67	0.2669	0.2612	+ .30
45	25	0.4730	0.4744	— .07	93,94	0.5818	0.5840	— .12
	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Hays			Schl.	Hays	
15	63,64	0.7105	0.7124	—0.10	54,55	0.8412	0.8419	—0.03
15	63,64	0.7130	0.7108	+ .12	54,55	0.8408	0.8409	— .01
1	98,99	0.2198	0.2182	+ .08	20,21	0.3396	0.3376	+ .10
2	97,98	0.2161	0.2202	— .22	21,22	0.3398	0.3376	+ .12
5	85,86	0.3189	0.3158	+ .16	33,34	0.2370	0.2384	— .07
7	78,79	0.4000	0.3978	+ .12	40,41	0.1598	0.1588	+ .05
7A	74,75	0.3018	0.3010	+ .04	44,45	0.2538	0.2531	+ .04
19	59,60	0.8144	0.8172	— .15	58,59	0.7409	0.7334	+ .39
23A	55,56	0.1681	0.1626	+ .29	63,64	0.3889	0.3884	+ .03
31	51,52	0.6894	0.6871	+ .12	66,67	0.8675	0.8676	— .01
32	49,50	0.8098	0.8086	+ .06	68,69	0.7445	0.7426	+ .10
33	48,49	0.1569	0.1561	+ .04	70,71	0.3988	0.4004	— .08
34	47,48	0.5111	0.5140	— .15	71	0.5398	0.5425	— .15
35	45,46	0.5094	0.5122	— .15	73	0.5412	0.5445	— .18
36	44	0.5342	0.5275	+ .36	74,75	0.5245	0.5174	+ .38
37	43,44	0.8969	0.8980	— .05	74,75	0.6581	0.6594	— .07
38	40,41	0.2764	0.2762	+ .01	78,79	0.2790	0.2751	+ .21
39	39,40	0.5344	0.5344	.00	79	0.5228	0.5226	+ .01
40	37,38	0.6996	0.6971	+ .13	80,81	0.8536	0.8578	— .21
43	29,30	0.8098	0.8078	+ .11	88,89	0.7468	0.7445	+ .13
44	29,30	0.2696	0.2618	+ .41	89,90	0.2901	0.2881	+ .11
45	25	0.4734	0.4758	— .12	93,94	0.5821	0.5852	— .16

TABLE III.—(Continued.) PLATE II: y MEASUREMENTS.

Star.	y direct.				y reversed.			
	Line.	$\frac{1}{2}m$, or Scale minus Star.		$K-H$	Line.	$\frac{1}{2}m$, or Scale minus Star.		$K-H$
		Kretz.	Hays.			Kretz.	Hays.	
15	51	0.4148	0.4171	—0.12	68	0.1352	0.1344	+0.04
15				"	68	0.1355	0.1368	— .07
2	48	0.0102	0.0088	+ .07	71	0.5406	0.5432	— .14
4	16	0.3411	0.3412	— .01	103	0.2192	0.2192	.00
6	65	0.6206	0.6175	+ .16	53	0.9330	0.9344	— .08
8	66	0.3665	0.3671	— .03	53	0.1856	0.1838	+ .10
10	44	0.3996	0.3972	+ .13	75	0.1539	0.1514	+ .13
11	38	0.6644	0.6645	— .01	80	0.8918	0.8906	+ .07
14	72	0.3570	0.3578	— .04	47	0.1970	0.1950	+ .11
16	17	0.1786	0.1791	— .03	102	0.3780	0.3768	+ .06
17	35	0.2662	0.2641	+ .11	84	0.2851	0.2858	— .04
18	36	0.1135	0.1128	+ .04	83	0.4404	0.4381	+ .12
20	35	0.3360	0.3325	+ .19	84	0.2159	0.2160	— .01
22	67	0.0880	0.0889	— .05	52	0.4618	0.4609	+ .05
23	64	0.6105	0.6099	+ .03	54	0.9411	0.9408	+ .03
24	34	0.7494	0.7484	+ .06	84	0.8025	0.8048	— .13
25	45	0.2892	0.2860	+ .17	74	0.2620	0.2612	+ .04
26	22	0.3085	0.3072	+ .07	97	0.2479	0.2501	— .11
28	55	0.3882	0.3879	+ .02	64	0.1601	0.1598	+ .02
29	79	0.0066	0.0085	— .10	40	0.5452	0.5449	+ .02
33	14	0.1540	0.1538	+ .01	105	0.4014	0.3988	+ .14
15	51	0.4176	0.4178	—0.01	68	0.1374	0.1369	+0.03
15	51	0.4146	0.4166	— .11	68	0.1348	0.1381	— .17
1	67	0.1062	0.1050	+ .06	52	0.4448	0.4442	+ .03
5	80	0.3721	0.3734	— .07	39	0.1791	0.1778	+ .07
7	32	0.3801	0.3786	+ .08	87	0.1742	0.1741	+ .01
7A	47	0.6575	0.6584	— .04	71	0.8970	0.8958	+ .07
19	77	0.3065	0.3075	— .05	42	0.2455	0.2440	+ .08
23A	66	0.1972	0.1936	+ .19	53	0.3584	0.3549	+ .19
27	44	0.1530	0.1530	.00	75	0.4005	0.3972	+ .17
31	35	0.6549	0.6524	+ .13	83	0.9032	0.9024	+ .05
32	11	0.8212	0.8249	— .21	107	0.7351	0.7302	+ .26
33	14	0.1548	0.1532	+ .08	105	0.4031	0.4030	+ .01
34	47	0.5749	0.5725	+ .13	71	0.9784	0.9769	+ .10
35	84	0.8458	0.8426	+ .16	34	0.7100	0.7115	— .08
36	62	0.2885	0.2879	+ .03	57	0.2629	0.2646	— .09
37	38	0.1672	0.1659	+ .07	81	0.3884	0.3875	+ .05
38	55	0.1725	0.1746	— .11	64	0.3778	0.3762	+ .08
39	61	0.4712	0.4724	— .06	58	0.0776	0.0781	— .03
40	16	0.5089	0.5050	+ .21	103	0.0502	0.0514	— .06
43	58	0.3500	0.3474	+ .14	61	0.2022	0.2029	— .04
44	77	0.7740	0.7744	— .02	41	0.7819	0.7749	+ .37
45	26	0.9096	0.9109	— .06	92	0.6474	0.6468	+ .03

TABLE III.—(Continued.) PLATE III: α MEASUREMENTS.

Star.	α direct.				α reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	56	0.3911	0.3919	—0.04	62,63	0.6618	0.6571	+0.25
2	89,90	0.4000	0.3988	+ .06	29	0.6560	0.6566	— .04
3	88,89	0.2970	0.2958	+ .06	30,31	0.2614	0.2550	+ .34
4	86,87	0.5528	0.5500	+ .16	32	0.5092	0.5078	+ .06
6	73,74	0.3035	0.3024	+ .06	45,46	0.2494	0.2486	+ .04
8	66,67	0.4601	0.4549	+ .28	52	0.5961	0.5900	+ .31
10	65,66	0.5760	0.5751	+ .06	53	0.4779	0.4745	+ .17
11	64,65	0.7194	0.7164	+ .15	53,54	0.8316	0.8298	+ .10
14	56	0.5784	0.5782	+ .01	62,63	0.4718	0.4712	+ .04
16	55	0.4950	0.4929	+ .11	63,64	0.5551	0.5549	+ .02
17	53,54	0.5736	0.5715	+ .10	65	0.4799	0.4765	+ .19
18	52	0.5315	0.5326	— .05	66,67	0.5189	0.5174	+ .07
20	51,52	0.6865	0.6869	— .02	67	0.3662	0.3678	— .08
22	49,50	0.7265	0.7282	— .08	68,69	0.8251	0.8240	+ .05
23	48,49	0.3615	0.3651	— .19	70,71	0.1871	0.1900	— .16
24	46,47	0.2424	0.2431	— .04	72,73	0.3086	0.3092	— .03
26	45,46	0.5581	0.5551	+ .15	73	0.4976	0.4960	+ .10
28	45	0.5479	0.5492	— .07	73,74	0.5059	0.5044	+ .08
29	44,45	0.4485	0.4481	+ .02	74	0.6046	0.6075	— .15
45	17,18	0.1620	0.1652	— .18	101,102	0.3970	0.3919	+ .28
		Kretz.	Hays.	$K-H$		Kretz.	Hays.	$K-H$
15	56	0.3914	0.3921	—0.04	62,63	0.6609	0.6506	+0.01
1	90,91	0.3850	0.3879	— .15	28,29	0.1676	0.1706	— .15
2	89,90	0.4021	0.3982	+ .19	29,30	0.1490	0.1516	— .15
5	77,78	0.4799	0.4805	— .03	41	0.5679	0.5720	— .21
7	70,71	0.5824	0.5838	— .07	48	0.4679	0.4664	+ .08
23A	47,48	0.3338	0.3400	— .33	71,72	0.2149	0.2169	— .12
25	45,46	0.6594	0.6636	— .23	73	0.3872	0.3941	— .37
27	45,46	0.2888	0.2896	— .04	73,74	0.2650	0.2638	+ .07
31	44	0.3776	0.3795	— .10	74,75	0.6784	0.6780	+ .03
32	42	0.5044	0.5028	+ .09	76,77	0.5489	0.5510	— .11
33	40,41	0.3505	0.3565	— .31	78,79	0.2052	0.2025	+ .15
34	39,40	0.6875	0.6896	— .12	79	0.3648	0.3654	— .04
35	37,38	0.6701	0.6732	— .16	81	0.3848	0.3850	— .01
36	36,37	0.2011	0.2008	+ .01	82,83	0.3482	0.3489	— .04
37	36	0.5816	0.5810	+ .03	82,83	0.4754	0.4729	+ .13
39	31,32	0.7004	0.7045	— .22	87	0.3522	0.3499	+ .12
40	30	0.3930	0.3948	— .11	88,89	0.6641	0.6649	— .05
43	22	0.4799	0.4785	+ .07	96,97	0.5762	0.5755	+ .03
44	21,22	0.4258	0.4301	— .24	97	0.6301	0.6301	.00
45	17,18	0.1648	0.1620	+ .14	101,102	0.3924	0.3916	+ .04

TABLE III.—(Continued.) PLATE III: y MEASUREMENTS.

y direct.					y reversed.				
Star.	Line.	$\frac{1}{2} m$, or Scale minus Star.		S—H	Line.	$\frac{1}{2} m$, or Scale minus Star.		S—H	
		Schl.	Hays.			Schl.	Hays.		
15	53	0.7856	0.7866	—0.06	65	0.7708	0.7681	+0.11	
2	50	0.3928	0.3918	+ .05	69	0.1626	0.1632	— .04	
3	35	0.8020	0.8015	+ .03	83	0.7534	0.7555	— .11	
4	18	0.7200	0.7194	+ .03	100	0.8375	0.8408	— .17	
6	67	0.9966	0.9959	+ .05	51	0.5574	0.5572	+ .01	
8	68	0.7408	0.7428	— .10	50	0.8144	0.8135	+ .05	
10	46	0.7719	0.7692	+ .14	72	0.7849	0.7830	+ .10	
11	41	0.0348	0.0328	+ .11	78	0.5232	0.5200	+ .16	
14	74	0.7250	0.7204	+ .24	44	0.8316	0.8282	+ .17	
16	19	0.5458	0.5422	+ .18	100	0.0146	0.0119	+ .14	
17	37	0.6339	0.6324	+ .08	81	0.9212	0.9208	+ .03	
18	38	0.4796	0.4758	+ .21	81	0.0778	0.0741	+ .20	
20	37	0.7001	0.7011	— .05	81	0.8534	0.8512	+ .12	
22	69	0.4481	0.4520	— .20	50	0.1001	0.0995	+ .04	
23	66	0.9716	0.9748	— .18	52	0.5775	0.5795	— .11	
24	37	0.1121	0.1108	+ .07	82	0.4415	0.4392	+ .13	
25	47	0.6516	0.6499	+ .10	71	0.9010	0.8996	+ .08	
26	24	0.6665	0.6665	.00	94	0.8871	0.8880	— .04	
28	57	0.7496	0.7475	+ .11	61	0.8059	0.8040	+ .10	
29	81	0.3720	0.3685	+ .18	38	0.1829	0.1800	+ .15	
33	16	0.5105	0.5121	— .10	103	0.0479	0.0425	+ .28	
15	53	0.7829	0.7849	—0.10	65	0.7679	0.7654	+0.14	
1	69	0.4875	0.4878	— .03	50	0.0631	0.0631	.00	
5	82	0.7499	0.7450	+ .25	36	0.8035	0.8031	+ .02	
7	34	0.7538	0.7535	+ .02	84	0.8054	0.8001	+ .29	
23A	68	0.5646	0.5611	+ .19	50	0.9911	0.9874	+ .21	
27	46	0.5160	0.5145	+ .08	73	0.0391	0.0369	+ .12	
31	38	0.0155	0.0181	— .14	81	0.5379	0.5371	+ .04	
32	14	0.1861	0.1812	+ .25	105	0.3760	0.3750	+ .05	
33	16	0.5112	0.5112	.00	103	0.0484	0.0438	+ .24	
34	49	0.9341	0.9330	+ .05	69	0.6175	0.6154	+ .11	
35	87	0.2068	0.2039	+ .15	32	0.3486	0.3469	+ .08	
36	64	0.6476	0.6454	+ .12	54	0.9068	0.9066	+ .02	
37	40	0.5184	0.5215	— .16	79	0.0340	0.0320	+ .11	
39	63	0.8299	0.8256	+ .22	55	0.7232	0.7225	+ .03	
40	18	0.8654	0.8618	+ .20	100	0.6912	0.6911	+ .01	
43	60	0.7026	0.7032	— .03	58	0.8524	0.8500	+ .13	
44	80	0.1296	0.1265	+ .17	39	0.4255	0.4234	+ .12	
45	29	0.2606	0.2582	+ .13	90	0.2991	0.2939	+ .28	

TABLE III.—(Continued.) PLATE IV: x MEASUREMENTS.

Star.	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	58,59	0.1086	0.1090	—0.02	60,61	0.4401	0.4376	+0.13
15	58,59	0.1098	0.1102	—0.02	60,61	0.4404	0.4382	+0.12
2	91,92	0.6132	0.6136	—0.02	27	0.4422	0.4411	+0.06
3	90,91	0.4950	0.4922	+0.15	28	0.5600	0.5594	+0.04
4	88,89	0.7311	0.7309	+0.01	29 30	0.8258	0.8249	+0.05
6	75,76	0.5378	0.5355	+0.12	43	0.5160	0.5156	+0.02
8	68,69	0.7006	0.6994	+0.06	50	0.3525	0.3500	+0.13
10	67,68	0.7929	0.7919	+0.06	50,51	0.7615	0.7609	+0.03
11	67	0.4274	0.4251	+0.12	51,52	0.6255	0.6258	—0.02
14	58,59	0.3225	0.3204	+0.11	60,61	0.2276	0.2272	+0.02
16	57,58	0.1772	0.1769	+0.02	61,62	0.3724	0.3735	—0.06
17	55,56	0.7785	0.7760	+0.13	62,63	0.7741	0.7760	—0.09
18	54 55	0.2394	0.2390	+0.02	64,65	0.3106	0.3105	+0.01
20	54	0.3919	0.3916	+0.02	64,65	0.6571	0.6581	—0.05
22	52	0.4664	0.4679	—0.08	66,67	0.5841	0.5841	0.00
23	50,51	0.5970	0.5962	+0.04	68	0.4560	0.4542	+0.10
24	48,49	0.4505	0.4506	—0.01	70	0.6004	0.6036	—0.17
25	48	0.3772	0.3728	+0.23	70,71	0.6758	0.6746	+0.06
26	47,48	0.7520	0.7505	+0.07	70,71	0.8045	0.8032	+0.08
27	47,48	0.5058	0.5064	—0.03	71	0.5465	0.5438	+0.15
28	47,48	0.2779	0.2762	+0.08	71,72	0.2745	0.2740	+0.03
29	46,47	0.7029	0.7000	+0.15	72	0.3510	0.3495	+0.08
45	19,20	0.3636	0.3658	—0.12	99,100	0.1926	0.1918	+0.04
15	58	0.6106	0.6114	—0.03	60,61	0.4404	0.4411	—0.03
15	58	0.6108	0.6094	+0.06	60,61	0.4415	0.4379	+0.19
1	92,93	0.6165	0.6235	—0.37	26	0.4415	0.4395	+0.10
2	91,92	0.6136	0.6129	+0.04	27	0.4419	0.4405	+0.07
5	79,80	0.7375	0.7375	0.00	38,39	0.8199	0.8202	—0.01
7	72,73	0.7816	0.7840	—0.12	45,46	0.7742	0.7744	—0.02
7A	68,69	0.6918	0.6942	—0.13	50	0.3619	0.3602	+0.09
19	54,55	0.2284	0.2284	0.00	64 65	0.3215	0.3205	+0.05
23A	49,50	0.5751	0.5765	—0.07	69	0.4772	0.4790	—0.10
31	46	0.5839	0.5828	+0.05	72,73	0.4711	0.4691	+0.11
32	44,45	0.1821	0.1831	—0.05	74,75	0.3680	0.3692	—0.06
33	42,43	0.5291	0.5280	+0.05	76	0.5155	0.5175	—0.11
34	42	0.4086	0.4081	+0.03	76,77	0.6442	0.6416	+0.14
35	40	0.4341	0.4365	—0.13	78,79	0.6210	0.6191	+0.10
36	38,39	0.4386	0.4406	—0.11	80	0.6144	0.6154	—0.06
37	38,39	0.2875	0.2868	+0.04	80,81	0.2640	0.2649	—0.05
38	34,35	0.6826	0.6850	—0.12	84	0.3724	0.3726	—0.02
39	34	0.4401	0.4415	—0.07	84,85	0.6141	0.6149	—0.04
40	32	0.5829	0.5836	—0.03	86,87	0.4729	0.4742	—0.07
43	24,25	0.2176	0.2154	+0.12	94,95	0.3385	0.3400	—0.08
44	23,24	0.6931	0.6961	—0.15	95	0.3649	0.3628	+0.12
45	19,20	0.3628	0.3605	+0.12	99,100	0.1922	0.1905	+0.09

TABLE III.—(Continued.) PLATE IV: γ MEASUREMENTS.

Star.	γ direct.				γ reversed.			
	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	54	0.3544	0.3561	—0.09	65	0.1952	0.1934	+0.10
15	54	0.3545	0.3578	— .17	65	0.1926	0.1939	— .07
2	50	0.9360	0.9344	+ .07	68	0.6195	0.6199	— .02
3	36	0.3470	0.3464	+ .03	83	0.2059	0.2051	+ .04
4	19	0.2672	0.2691	— .10	100	0.2875	0.2898	— .12
6	68	0.5526	0.5525	+ .01	50	0.9996	1.0015	— .12
8	69	0.2996	0.3001	— .03	50	0.2510	0.2481	+ .15
10	47	0.3356	0.3349	+ .03	72	0.2148	0.2164	— .08
11	41	0.5992	0.5962	+ .16	77	0.9564	0.9561	+ .03
14	75	0.2926	0.2945	— .10	44	0.2558	0.2565	— .04
16	20	0.1172	0.1185	— .07	99	0.4378	0.4361	+ .09
17	38	0.2076	0.2078	— .01	81	0.3446	0.3442	+ .02
18	39	0.0536	0.0534	+ .01	80	0.4979	0.4991	— .06
20	38	0.2798	0.2785	+ .07	81	0.2716	0.2736	— .11
22	70	0.0256	0.0255	+ .01	49	0.5252	0.5246	+ .03
23	67	0.5522	0.5525	— .02	52	— .0032	0.0002	— .18
24	37	0.6928	0.6925	+ .02	81	0.8572	0.8588	— .10
25	48	0.2345	0.2330	+ .08	71	0.3155	0.3162	— .04
26	25	0.2515	0.2508	+ .04	94	0.3014	0.3042	— .15
28	58	0.3315	0.3305	+ .05	61	0.2180	0.2182	— .01
29	81	0.9495	0.9505	— .06	37	0.6024	0.6021	+ .02
33	17	0.1019	0.0998	+ .11	102	0.4535	0.4555	— .11
15	54	0.3538	0.3568	—0.15	65	0.1951	0.1922	+0.15
15	54	0.3562	0.3579	— .08	65	0.1929	0.1904	+ .13
1	70	0.0282	0.0304	— .12	49	0.5239	0.5224	+ .07
5	83	0.2979	0.2979	.00	36	0.2529	0.2540	— .06
7	35	0.3186	0.3182	+ .02	84	0.2356	0.2362	— .03
7A	50	0.5930	0.5936	— .03	68	0.9646	0.9656	— .07
19	80	0.2471	0.2455	+ .08	39	0.3060	0.3088	— .15
23A	69	0.1386	0.1401	— .08	50	0.4115	0.4120	— .03
27	47	0.0961	0.0966	— .03	72	0.4552	0.4564	— .06
31	38	0.5966	0.5961	+ .02	80	0.9584	0.9589	— .04
32	14	0.7720	0.7728	— .04	104	0.7861	0.7886	— .13
33	17	0.1015	0.1024	— .05	102	0.4555	0.4562	— .04
34	50	0.5242	0.5254	— .06	69	0.0256	0.0276	— .11
35	87	0.7922	0.7922	.00	31	0.7645	0.7664	— .10
36	65	0.2356	0.2331	+ .13	54	0.3152	0.3154	— .01
37	41	0.1111	0.1115	— .02	78	0.4406	0.4410	— .02
38	58	0.1198	0.1186	+ .06	61	0.4311	0.4320	— .05
39	64	0.4198	0.4202	— .02	55	0.1292	0.1296	— .02
40	19	0.4612	0.4622	— .05	100	0.0895	0.0895	.00
43	61	0.2991	0.3002	— .06	58	0.2488	0.2490	— .01
44	80	0.7300	0.7276	+ .13	38	0.8281	0.8304	— .13
45	29	0.8686	0.8679	+ .03	89	0.6874	0.6874	.00

TABLE III.—(Continued.) PLATE V: x MEASUREMENTS.

Star.	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	55,56	0.2626	0.2598	+0.15	63,64	0.3082	0.3041	+0.22
15	55,56	0.2629	0.2598	+ .16	63,64	0.3078	0.3055	+ .12
2	88,89	0.7681	0.7676	+ .03	29,30	0.8032	0.8031	+ .01
3	87,88	0.6692	0.6708	— .08	30,31	0.9018	0.9005	+ .06
4	85,86	0.9279	0.9281	— .02	32,33	0.6450	0.6451	— .01
6	72,73	0.6664	0.6684	— .11	45,46	0.8999	0.9011	— .07
8	65,66	0.8274	0.8279	— .03	52,53	0.7400	0.7415	— .08
10	64,66	0.4472	0.4454	+ .10	53,54	0.6195	0.6194	+ .01
11	64,65	0.0944	0.0921	+ .12	54,55	0.4770	0.4764	+ .03
14	55,56	0.4405	0.4400	+ .03	63,64	0.1269	0.1251	+ .10
16	54,55	0.3751	0.3738	+ .07	64,65	0.1938	0.1938	.00
17	52,54	0.4486	0.4476	+ .05	65,66	0.6221	0.6204	+ .09
18	51,52	0.4046	0.4030	+ .08	67,68	0.1645	0.1631	+ .07
20	51,52	0.0611	0.0592	+ .10	67,68	0.5110	0.5089	+ .11
22	49,50	0.0939	0.0929	+ .05	69,70	0.4755	0.4735	+ .11
23	47,48	0.7248	0.7234	+ .07	70,71	0.8448	0.8422	+ .14
24	45,46	0.6199	0.6192	+ .04	72,74	0.4516	0.4486	+ .17
25	45,46	0.0336	0.0326	+ .05	73,74	0.5389	0.5368	+ .11
26	44,46	0.4384	0.4380	+ .02	73,74	0.6342	0.6314	+ .15
27	44,45	0.6610	0.6584	+ .14	73,74	0.9106	0.9106	.00
28	44,45	0.4170	0.4146	+ .13	74,75	0.1540	0.1514	+ .14
29	43,44	0.8122	0.8128	— .03	74,75	0.7590	0.7554	+ .19
45	16,17	0.5412	0.5450	— .20	102,103	0.0298	0.0298	.00
15	55,56	0.2614	0.2596	+0.10	63,64	0.3075	0.3050	+0.13
15	55,56	0.2590	0.2598	— .04	63,64	0.3069	0.3049	+ .11
1	89,90	0.7531	0.7504	+ .14	28,29	0.8219	0.8161	+ .31
2	88,89	0.7692	0.7650	+ .22	29,30	0.8012	0.8020	— .05
5	76,77	0.8410	0.8396	+ .08	41,42	0.7296	0.7272	+ .13
7	69,71	0.4560	0.4555	+ .02	48,49	0.6126	0.6108	+ .10
19	51,52	0.3436	0.3415	+ .11	67,68	0.2275	0.2262	+ .07
23A	46,47	0.7032	0.7029	+ .02	71,72	0.8665	0.8656	+ .04
31	43,44	0.2522	0.2494	+ .15	75,76	0.3184	0.3155	+ .15
32	41,42	0.3892	0.3901	— .05	77,78	0.1858	0.1832	+ .14
34	39,40	0.0630	0.0658	— .15	79,80	0.5071	0.5039	+ .17
35	37,38	0.0308	0.0279	+ .15	81,82	0.5371	0.5326	+ .24
37	35,36	0.4526	0.4510	+ .08	83,84	0.1158	0.1126	+ .17
39	31,32	0.0745	0.0725	+ .11	87,88	0.4980	0.4962	+ .10
40	29,30	0.2818	0.2772	+ .24	89,90	0.2950	0.2902	+ .25
43	21,22	0.3564	0.3550	+ .07	97,98	0.2196	0.2159	+ .20
44	20,21	0.7966	0.7962	+ .02	97,98	0.7794	0.7735	+ .31
45	16,17	0.5421	0.5444	— .12	102,103	0.0304	0.0251	+ .28

TABLE III.—(Continued.) PLATE V: y MEASUREMENTS.

Star.	y direct.				y reversed.			
	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	54	0.7016	0.7006	+0.05	64	0.8646	0.8650	—0.02
15	54	0.6999	0.7008	— .05	64	0.8626	0.8652	— .14
2	51	0.3171	0.3152	+ .10	68	0.2526	0.2556	— .16
3	36	0.7258	0.7270	— .06	82	0.8448	0.8426	+ .11
4	19	0.6431	0.6422	+ .05	99	0.9318	0.9300	+ .08
6	68	0.9181	0.9175	+ .02	50	0.6506	0.6496	+ .05
8	69	0.6558	0.6568	— .05	49	0.9089	0.9072	+ .10
10	47	0.6930	0.6931	— .01	71	0.8759	0.8748	+ .06
11	41	0.9518	0.9546	— .13	77	0.6155	0.6144	+ .06
14	75	0.6389	0.6386	+ .02	43	0.9291	0.9300	— .04
16	20	0.4612	0.4616	— .02	99	0.1106	0.1068	+ .20
17	38	0.5500	0.5489	+ .06	81	0.0182	0.0166	+ .08
18	39	0.3969	0.3975	— .03	80	0.1698	0.1715	— .09
20	38	0.6178	0.6168	+ .05	80	0.9494	0.9476	+ .08
22	70	0.3664	0.3648	+ .08	49	0.2002	0.2016	— .07
23	67	0.8894	0.8892	+ .02	51	0.6755	0.6768	— .07
24	38	0.0304	0.0291	+ .07	81	0.5392	0.5402	— .05
25	48	0.5638	0.5661	— .12	71	— .0008	— .0005	— .02
26	25	0.5840	0.5829	+ .06	94	— .0120	— .0119	— .01
27	47	0.4309	0.4311	— .01	72	0.1349	0.1350	— .01
28	58	0.6628	0.6626	+ .01	60	0.9031	0.9042	— .05
29	82	0.2832	0.2811	+ .11	37	0.2871	0.2846	+ .13
15	54	0.7029	0.7028	+0.01	64	0.8622	0.8644	—0.13
15	54	0.7035	0.7011	+ .13	64	0.8644	0.8636	+ .03
1	70	0.4172	0.4164	+ .04	49	0.1516	0.1570	— .29
5	83	0.6724	0.6675	+ .26	35	0.8962	0.8986	— .12
7	35	0.6772	0.6771	+ .01	83	0.8942	0.8971	— .14
19	80	0.5896	0.5870	+ .14	39	— .0152	— .0150	— .01
23A	69	0.4802	0.4760	+ .22	50	0.0929	0.0920	+ .05
31	38	0.9318	0.9305	+ .08	80	0.6404	0.6350	+ .29
32	15	0.1042	0.1020	+ .12	104	0.4722	0.4699	+ .12
34	50	0.8502	0.8475	+ .15	68	0.7165	0.7156	+ .05
35	88	0.1154	0.1148	+ .03	31	0.4542	0.4576	— .18
37	41	0.4381	0.4368	+ .07	78	0.1349	0.1339	+ .05
39	64	0.7380	0.7364	+ .08	54	0.8279	0.8278	+ .01
40	19	0.7798	0.7792	+ .03	99	0.7938	0.7948	— .06
43	61	0.6096	0.6090	+ .03	57	0.9594	0.9584	+ .04
44	81	0.0361	0.0339	+ .12	38	0.5328	0.5342	— .07
45	30	0.1700	0.1699	+ .01	89	0.3978	0.4001	— .13

TABLE III.—(Continued.) PLATE VII: α MEASUREMENTS.

Star.	α direct.				α reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-H$	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-H$
		Schl.	Hays.			Schl.	Hays.	
15	61,62	0.6390	0.6438	—0.25	56,58	0.4129	0.4134	—0.03
2	95,96	0.1465	0.1489	— .13	23,24	0.4122	0.4078	+ .23
3	93,95	0.5530	0.5496	+ .17	24,25	0.5062	0.5049	+ .07
4	92,93	0.3122	0.3115	+ .04	26,27	0.2438	0.2468	— .16
6	78,80	0.5484	0.5484	.00	39,40	0.5079	0.5094	— .08
8	72,73	0.2074	0.2076	— .01	46,47	0.3484	0.3449	+ .18
10	71,72	0.3296	0.3339	— .23	47,48	0.2242	0.2252	— .05
11	70,71	0.4775	0.4796	— .11	47,49	0.5779	0.5730	+ .25
14	61,62	0.8200	0.8219	— .10	56,57	0.7348	0.7306	+ .22
16	60,61	0.7609	0.7619	— .05	57,58	0.7938	0.7931	+ .03
17	59,60	0.3314	0.3268	+ .24	59,60	0.2208	0.2222	— .07
18	57,58	0.7879	0.7915	— .20	60,61	0.7640	0.7655	— .07
20	57,58	0.4450	0.4424	+ .14	60,62	0.6079	0.6100	— .10
22	55,56	0.4754	0.4781	— .14	62,64	0.5755	0.5742	+ .07
23	54,55	0.1081	0.1092	— .06	64,65	0.4446	0.4468	— .11
24	51,53	0.5110	0.5112	— .01	66,67	0.5442	0.5444	— .01
25	51,52	0.4162	0.4139	+ .12	67,68	0.1366	0.1356	+ .05
26	51,52	0.3231	0.3251	— .10	67,68	0.2321	0.2316	+ .03
27	50,52	0.5444	0.5456	— .06	67,68	0.5095	0.5095	.00
28	50,51	0.8028	0.8031	— .01	67,68	0.7540	0.7515	+ .13
29	50,51	0.1946	0.1925	+ .11	68,69	0.3584	0.3595	— .06
31	49,50	0.6379	0.6386	— .04	68,70	0.4170	0.4178	— .04
32	47,48	0.7781	0.7770	+ .06	70,71	0.7778	0.7756	+ .11
33	46,47	0.1226	0.1226	.00	72,73	0.4316	0.4350	— .18
45	22,24	0.4344	0.4370	— .13	95,96	0.6214	0.6179	+ .19
		Schl.	Kretz	$S-K$		Schl.	Kretz.	$S-K$
15	61,62	0.6420	0.6444	—0.12	56,58	0.4105	0.4090	+0.08
I	96,97	0.1244	0.1286	— .22	22,23	0.4282	0.4268	+ .07
2	95,96	0.1474	0.1480	— .03	23,24	0.4101	0.4068	+ .17
5	83,84	0.2168	0.2158	+ .05	35,36	0.3389	0.3389	.00
7	76,77	0.3461	0.3411	+ .26	42,43	0.2086	0.2081	+ .03
7A	72,73	0.2332	0.2351	— .10	46,47	0.3231	0.3216	+ .08
19	57,58	0.7251	0.7224	+ .14	60,61	0.8312	0.8338	— .14
21	56,58	0.4262	0.4250	+ .06	61,62	0.6336	0.6280	+ .30
23A	52,54	0.5909	0.5854	+ .28	65,66	0.4660	0.4674	— .07
34	45,46	0.4486	0.4469	+ .09	73,74	0.1078	0.1046	+ .17
35	43,44	0.4162	0.4164	— .01	75,76	0.1359	0.1390	— .16
36	41,43	0.4652	0.4626	+ .14	76,77	0.5950	0.5931	+ .10
37	41,42	0.8426	0.8446	— .12	76,77	0.7156	0.7064	+ .49
38	38,39	0.2091	0.2134	— .23	80,81	0.3486	0.3459	+ .14
39	37,38	0.4620	0.4608	+ .06	80,82	0.5946	0.5924	+ .12
40	35,36	0.6735	0.6731	+ .02	82,84	0.3860	0.3824	+ .19
41	31,32	0.6124	0.6054	+ .37	86,88	0.4504	0.4526	— .12
42	29,30	0.6276	0.6230	+ .24	88,90	0.4315	0.4300	+ .07
43	27,28	0.7438	0.7431	+ .04	90,91	0.8136	0.8128	+ .03
44	27,28	0.1826	0.1789	+ .20	91,92	0.3741	0.3744	— .02
45	22,24	0.4362	0.4340	+ .11	93,96	0.6239	0.6222	+ .09

TABLE III.—(Continued.) PLATE VII: y MEASUREMENTS.

y direct.					y reversed.				
Star.	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Line.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	
		Schl.	Kretz.			Schl.	Kretz.		
15	62	0.0196	0.0186	+ .05	57	0.5301	0.5302	— .01	
2	58	0.6448	0.6446	+ .01	60	0.9095	0.9045	+ .28	
3	44	0.0536	0.0534	+ .01	75	0.4972	0.4942	+ .16	
4	26	0.9754	0.9744	+ .07	92	0.5794	0.5774	+ .11	
6	76	0.2415	0.2414	+ .01	43	0.3114	0.3072	+ .22	
8	76	0.9804	0.9799	+ .02	42	0.5731	0.5720	+ .06	
10	55	0.0164	0.0149	+ .08	64	0.5348	0.5346	+ .01	
11	49	0.2788	0.2782	+ .03	70	0.2735	0.2714	+ .11	
14	82	0.9624	0.9610	+ .06	36	0.5922	0.5928	— .03	
16	27	0.7882	0.7840	+ .23	91	0.7689	0.7664	+ .13	
17	45	0.8736	0.8712	+ .14	73	0.6788	0.6784	+ .02	
18	46	0.7182	0.7169	+ .07	72	0.8365	0.8336	+ .16	
20	45	0.9394	0.9379	+ .10	73	0.6142	0.6149	— .04	
22	77	0.6870	0.6864	+ .03	41	0.8685	0.8691	— .04	
23	75	0.2095	0.2090	+ .03	44	0.3416	0.3448	— .16	
24	45	0.3472	0.3462	+ .05	74	0.2028	0.2011	+ .09	
25	55	0.8854	0.8852	+ .02	63	0.6631	0.6628	+ .02	
26	32	0.9052	0.9058	— .02	86	0.6512	0.6496	+ .08	
27	54	0.7488	0.7494	— .03	64	0.8004	0.7996	+ .05	
28	65	0.9808	0.9806	+ .01	53	0.5726	0.5720	+ .03	
29	89	0.6049	0.6022	+ .14	29	0.9515	0.9535	— .12	
31	46	0.2505	0.2515	— .05	73	0.3002	0.3026	— .13	
32	22	0.4230	0.4215	+ .08	97	0.1355	0.1334	+ .11	
33	24	0.7514	0.7492	+ .11	94	0.8048	0.8038	+ .05	
		Kretz.	Hays.	$K-H$		Kretz.	Hays.	$K-H$	
15	62	0.0182	0.0246	— .034	57	0.5318	0.5329	— .06	
1	77	0.7418	0.7462	— .23	41	0.8111	0.8120	— .04	
5	90	0.9979	0.9966	+ .08	28	0.5592	0.5605	— .06	
7	43	— .0005	— .0001	— .02	76	0.5508	0.5521	— .06	
7A	58	0.2669	0.2690	— .11	61	0.2860	0.2868	— .04	
19	87	0.9085	0.9064	+ .10	31	0.6464	0.6486	— .12	
21	86	0.3856	0.3829	+ .14	33	0.1770	0.1775	— .02	
23A	76	0.7939	0.7956	— .08	42	0.7582	0.7600	— .10	
33	24	0.7534	0.7530	+ .02	94	0.8038	0.8059	— .12	
34	58	0.1652	0.1675	— .12	61	0.3841	0.3861	— .11	
35	95	0.4320	0.4320	.00	24	0.1251	0.1244	+ .04	
36	72	0.8776	0.8750	+ .14	46	0.6749	0.6749	.00	
37	48	0.7515	0.7538	— .12	70	0.8013	0.8045	— .17	
38	65	0.7550	0.7541	+ .05	53	0.8013	0.8026	— .07	
39	72	0.0530	0.0519	+ .06	47	0.5000	0.5019	— .10	
40	27	0.0934	0.0952	— .10	92	0.4606	0.4615	— .05	
41	54	0.4615	0.4634	— .10	65	0.0902	0.0882	+ .11	
42	54	0.3646	0.3646	.00	65	0.1885	0.1909	— .13	
43	68	0.9215	0.9211	+ .01	50	0.6322	0.6345	— .12	
44	88	0.3458	0.3438	+ .10	31	0.2065	0.2076	— .06	
45	37	0.4842	0.4869	— .14	82	0.0679	0.0672	+ .04	

TABLE III.—(Continued.) PLATE VIII: α MEASUREMENTS.

Star.	α direct.				α reversed.			
	Lines.	$\frac{1}{2} m$, or Scale <i>minus</i> Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale <i>minus</i> Star.		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	60.61	0.2671	0.2648	+ 0.12	58.59	0.2850	0.2819	+ 0.16
15					58.59	0.2851	0.2828	+ .12
2	93.94	0.7701	0.7728	— .14	24.25	0.7856	0.7825	+ .17
3	92.93	0.6742	0.6731	+ .06	25.27	0.3849	0.3842	+ .04
4	90.92	0.4280	0.4276	+ .02	27.28	0.6299	0.6285	+ .07
6	77.78	0.6768	0.6800	— .17	40.42	0.3745	0.3741	+ .02
8	70.72	0.3415	0.3375	+ .21	47.48	0.7154	0.7150	+ .02
10	69.71	0.4545	0.4509	+ .19	48.49	0.6002	0.5968	+ .18
11	68.70	0.6009	0.6011	— .01	49.50	0.4522	0.4505	+ .09
14	60.61	0.4509	0.4502	+ .04	57.59	0.5990	0.5988	+ .02
16	59.60	0.3821	0.3795	+ .14	59.60	0.1725	0.1725	.00
17	57.59	0.4562	0.4515	+ .25	60.61	0.5979	0.5978	+ .01
18	56.57	0.4119	0.4091	+ .15	62.63	0.1374	0.1385	— .06
20	55.57	0.5689	0.5650	+ .21	62.63	0.4841	0.4874	— .18
22	53.55	0.6064	0.6061	+ .01	64.65	0.4450	0.4465	— .08
23	52.53	0.7400	0.7380	+ .11	65.66	0.8161	0.8162	.00
24	50.51	0.6309	0.6234	+ .40	67.69	0.4269	0.4252	+ .09
25	49.51	0.5430	0.5401	+ .15	68.69	0.5071	0.5089	— .10
26	49.51	0.4416	0.4394	+ .12	68.69	0.6129	0.6110	+ .10
27	49.50	0.6729	0.6702	+ .14	68.70	0.3799	0.3794	+ .03
28	49.50	0.4262	0.4281	— .10	68.70	0.6234	0.6234	.00
29	48.49	0.8270	0.8258	+ .07	69.70	0.7252	0.7242	+ .05
31	48.49	0.2589	0.2556	+ .17	70.71	0.2974	0.2940	+ .18
45	21.22	0.5558	0.5545	+ .07	96.98	0.5000	0.4980	+ .10
		$\frac{1}{2} m$, or Scale <i>minus</i> Star.		$S-K$		$\frac{1}{2} m$, or Scale <i>minus</i> Star.		$S-K$
		Schl.	Hays.			Schl.	Hays.	
15	60.61	0.2681	0.2700	— 0.10	58.59	0.2839	0.2840	— 0.01
15	60.61	0.2664	0.2661	+ .02	58.59	0.2852	0.2856	— .02
1	94.95	0.7576	0.7529	+ .24	23.24	0.8044	0.7984	+ .32
2	93.94	0.7709	0.7736	— .14	24.25	0.7872	0.7855	+ .08
5	81.82	0.8511	0.8485	+ .13	36.37	0.7059	0.7065	— .03
7	74.76	0.4639	0.4646	— .03	43.44	0.5938	0.5896	+ .22
7A	70.72	0.3661	0.3594	+ .35	47.48	0.6921	0.6902	+ .10
19	56.57	0.3584	0.3575	+ .05	62.63	0.1961	0.1901	+ .31
23A	51.52	0.7201	0.7181	+ .10	66.67	0.8364	0.8356	+ .05
32	46.47	0.3970	0.3920	+ .26	72.73	0.1592	0.1631	— .21
33	44.45	0.7401	0.7400	+ .01	73.74	0.8152	0.8174	— .11
34	43.45	0.5712	0.5742	— .17	74.75	0.4800	0.4814	— .07
35	41.43	0.5529	0.5520	+ .04	76.77	9.5016	0.5000	+ .08
36	40.41	0.5946	0.5938	+ .04	77.79	0.4630	0.4615	+ .08
37	40.41	0.4676	0.4679	— .02	77.79	0.5885	0.5921	— .19
38	36.37	0.8402	0.8384	+ .08	81.82	0.7172	0.7172	.00
39	35.37	0.5885	0.5916	— .17	82.83	0.4624	0.4592	+ .17
40	34.35	0.2886	0.2882	+ .02	84.85	0.2649	0.2655	— .03
41	30.31	0.2381	0.2302	+ .42	88.89	0.3200	0.3199	+ .01
42	28.29	0.2592	0.2594	— .01	90.91	0.3016	0.2991	+ .13
43	26.27	0.3708	0.3731	— .12	82.93	0.1818	0.1814	+ .02
44	25.26	0.8192	0.8188	+ .03	92.93	0.7378	0.7345	+ .17
45	21.22	0.5601	0.5591	+ .05	96.98	0.5009	0.4981	+ .14

TABLE III.—(Continued.) PLATE VIII: y MEASUREMENTS.

Star.	y direct.				y reversed.			
	Line.	$\frac{1}{2}$ m. or Scale minus Star.		$K-H$	Line.	$\frac{1}{2}$ m. or Scale minus Star.		$K-H$
		Kretz.	Hays.			Kretz.	Hays.	
15	55	0.0068	0.0070	—0.01	64	0.5411	0.5416	—0.03
15					64	0.5421	0.5425	— .02
2	51	0.6239	0.6241	— .01	67	0.9258	0.9290	— .19
3	37	0.0348	0.0326	+ .12	82	0.5169	0.5194	— .13
4	19	0.9572	0.9590	— .08	99	0.5984	0.6002	— .10
6	69	0.2228	0.2200	+ .15	50	0.3288	0.3249	+ .21
8	69	0.9606	0.9614	— .03	49	0.5875	0.5898	— .12
10	48	—0.0029	0.0000	— .15	71	0.5505	0.5512	— .04
11	42	0.2579	0.2596	— .09	77	0.2900	0.2915	— .08
14	75	0.9466	0.9432	+ .16	43	0.6034	0.6072	— .20
16	20	0.7739	0.7724	+ .07	98	0.7801	0.7820	— .11
17	38	0.8599	0.8614	— .07	80	0.6928	0.6949	— .11
18	39	0.7045	0.7068	— .12	79	0.8506	0.8478	+ .15
20	38	0.9272	0.9284	— .05	80	0.6249	0.6264	— .08
22	70	0.6700	0.6721	— .11	48	0.8775	0.8769	+ .04
23	68	0.1906	0.1938	— .17	51	0.3525	0.3596	— .37
24	38	0.3356	0.3372	— .08	81	0.2140	0.2150	— .05
25	48	0.8722	0.8729	— .03	70	0.6748	0.6775	— .14
26	25	0.8939	0.8966	— .13	93	0.6598	0.6611	— .07
27	47	0.7358	0.7394	— .19	71	0.8111	0.8096	+ .08
28	58	0.9688	0.9678	+ .04	60	0.5792	0.5779	+ .07
29	82	0.5856	0.5869	— .07	36	0.9645	0.9650	— .04
31	39	0.2340	0.2400	— .32	80	0.3161	0.3192	— .16
33	17	0.7346	0.7341	+ .03	101	0.8170	0.8195	— .14
		Schl.	Kretz.	$S-K$		Schl.	Kretz.	$S-K$
15	55	0.0095	0.0072	+0.12	64	0.5411	0.5409	+0.01
15	55	0.0105	0.0076	+ .15	64	0.5458	0.5418	+ .21
1	70	0.7176	0.7158	+ .09	48	0.8352	0.8331	+ .10
5	83	0.9740	0.9754	— .09	35	0.5774	0.5751	+ .12
7	36	—0.0154	—0.0178	+ .13	83	0.5680	0.5670	+ .05
7A	51	0.2508	0.2484	+ .13	68	0.3062	0.3025	+ .20
19	80	0.8938	0.8884	+ .28	38	0.6646	0.6609	+ .20
23A	69	0.7815	0.7792	+ .12	49	0.7719	0.7688	+ .16
32	15	0.4090	0.4109	— .10	104	0.1476	0.1456	+ .11
33	17	0.7384	0.7378	+ .03	101	0.8194	0.8171	+ .13
34	51	0.1562	0.1560	+ .01	68	0.3972	0.3955	+ .09
35	88	0.4174	0.4150	+ .13	31	0.1369	0.1330	+ .21
36	65	0.8636	0.8619	+ .10	53	0.6901	0.6880	+ .11
37	41	0.7460	0.7445	+ .03	77	0.8105	0.8101	+ .02
38	58	0.7471	0.7446	+ .13	60	0.8075	0.8049	+ .14
39	65	0.0412	0.0418	— .03	54	0.5091	0.5089	+ .01
40	20	0.0869	0.0869	.00	99	0.4692	0.4708	— .08
41	47	0.4586	0.4596	— .05	72	0.0951	0.0960	— .05
42	47	0.3562	0.3532	+ .16	72	0.2010	0.1981	+ .15
43	61	0.9196	0.9135	+ .33	57	0.6349	0.6349	.00
44	81	0.3348	0.3334	+ .07	38	0.2199	0.2176	+ .12
45	30	0.4771	0.4782	— .06	89	0.0745	0.0785	— .21

TABLE III.—(Continued.) PLATE IX: x MEASUREMENTS.

Star.	x direct.				x reversed.			
	Lines.	$\frac{1}{2} m$, or Scale minus Star.		$S-K$	Lines.	$\frac{1}{2} m$, or Scale minus Star		$S-K$
		Schl.	Kretz.			Schl.	Kretz.	
15	54,55	0.8900	0.8901	—0.01	63,64	0.6599	0.6628	—0.15
2	88,89	0.3924	0.3978	— .29	30,31	0.1621	0.1625	— .02
3	87,88	0.3025	0.3008	+ .09	31,32	0.2570	0.2544	+ .14
4	85,86	0.5701	0.5648	+ .28	32,34	0.4876	0.4918	— .22
6	72,73	0.2920	0.2899	+ .11	46,47	0.2621	0.2621	.00
8	65,66	0.4511	0.4461	+ .26	53,54	0.1020	0.1029	— .05
10	64,65	0.5751	0.5730	+ .11	53,55	0.4780	0.4776	+ .02
11	63,64	0.7320	0.7318	+ .01	54,55	0.8220	0.8205	+ .07
14	55,56	0.0608	0.0609	— .01	63,64	0.4905	0.4886	+ .10
16	53,55	0.5259	0.5236	+ .12	64,65	0.5291	0.5299	— .04
17	52,53	0.5848	0.5914	— .35	65,67	0.4656	0.4661	— .03
18	50,52	0.5451	0.5448	+ .01	67,68	0.5085	0.5105	— .11
20	50,51	0.6969	0.6962	+ .04	67,68	0.8598	0.8592	+ .04
22	48,49	0.7159	0.7158	+ .01	69,70	0.8362	0.8382	— .10
23	47,48	0.3495	0.3515	— .11	71,72	0.2048	0.2044	— .03
24	45,46	0.2512	0.2520	— .04	73,74	0.2981	0.2989	— .04
25	44,45	0.6609	0.6601	+ .04	73,74	0.8926	0.8914	+ .07
26	44,45	0.5798	0.5791	+ .04	73,75	0.4729	0.4739	— .05
27	44,45	0.2938	0.2900	+ .20	74,75	0.2616	0.2608	+ .04
28	43,45	0.5416	0.5436	— .11	74,75	0.5095	0.5086	+ .05
29	43,44	0.4295	0.4300	— .03	75,76	0.1226	0.1228	— .01
45	16,17	0.1870	0.1888	— .10	102,103	0.3682	0.3689	— .04
				$K-H$				
					Kretz.	Hays.	$K-H$	
15	54,55	0.8899	0.8898	+0.01	63,64	0.6629	0.6628	+0.01
1	89,90	0.3738	0.3658	+ .42	29,30	0.1809	0.1800	+ .05
2	88,89	0.3938	0.3946	— .04	30,31	0.1600	0.1602	— .01
5	76,77	0.4536	0.4546	— .05	42,43	0.0996	0.0986	+ .05
7	69,70	0.5980	0.5940	+ .21	48,50	0.4599	0.4625	— .13
23A	46,47	0.3264	0.3236	+ .14	72,73	0.2242	0.2258	— .08
31	42,43	0.8836	0.8880	— .22	75,76	0.6685	0.6692	— .04
32	40,42	0.5325	0.5338	— .07	77,78	0.5216	0.5194	+ .12
33	39,40	0.3811	0.3829	— .10	79,80	0.1719	0.1732	— .07
34	38,39	0.6959	0.6980	— .12	79,80	0.8624	0.8601	+ .13
35	36,37	0.6520	0.6511	+ .05	81,82	0.9028	0.9005	+ .13
36	35,36	0.2006	0.1999	+ .04	83,84	0.3488	0.3510	— .12
37	35,36	0.0874	0.0931	— .30	83,84	0.4661	0.4621	+ .21
38	31,32	0.4465	0.4495	— .16	87,88	0.1064	0.1052	+ .06
39	30,31	0.7034	0.6981	+ .28	87,88	0.8530	0.8550	— .10
40	28,29	0.9286	0.9256	+ .17	89,90	0.6300	0.6298	+ .01
43	20,22	0.4879	0.4882	— .01	97,98	0.5671	0.5681	— .05
44	20,21	0.4152	0.4138	+ .07	98,99	0.1415	0.1409	+ .03
45	16,17	0.1885	0.1902	— .09	102,103	0.3685	0.3682	+ .02

TABLE III.—(Concluded.) PLATE IX: y MEASUREMENTS.

y direct.					y reversed.				
Line.	Star.	$\frac{1}{2} m$, or Scale minus Star.		K—H	Line.	$\frac{1}{2} m$, or Scale minus Star.		K—H	
		Kretz.	Hays.			Kretz.	Hays.		
15	56	0.6574	0.6586	—0.06	62	0.8968	0.8969	—0.00	
2	53	0.2924	0.2885	+ .21	66	0.2601	0.2619	— .10	
3	38	0.7025	0.7022	+ .02	80	0.8546	0.8548	— .02	
4	21	0.6201	0.6211	— .05	97	0.9386	0.9390	— .03	
6	70	0.8824	0.8836	— .05	48	0.6685	0.6701	— .08	
8	71	0.6176	0.6160	+ .08	47	0.9336	0.9311	+ .12	
10	49	0.6535	0.6530	+ .02	69	0.8961	0.8986	— .12	
11	43	0.9194	0.9174	+ .10	75	0.6335	0.6356	— .11	
14	77	0.5956	0.5951	+ .03	41	0.9594	0.9579	+ .10	
16	22	0.4234	0.4211	+ .12	97	0.1318	0.1336	— .10	
17	40	0.5092	0.5088	+ .02	79	0.0488	0.0450	+ .20	
18	41	0.3556	0.3512	+ .23	78	0.2010	0.2028	— .10	
20	40	0.5775	0.5768	+ .04	78	0.9788	0.9775	+ .05	
22	72	0.3186	0.3181	+ .03	47	0.2341	0.2351	— .05	
23	69	0.8454	0.8441	+ .06	49	0.7089	0.7101	— .06	
24	39	0.9861	0.9831	+ .17	79	0.5705	0.5735	— .16	
25	50	0.5185	0.5192	— .04	69	0.0304	0.0294	+ .05	
26	27	0.5389	0.5426	— .20	92	0.0139	0.0171	— .17	
27	49	0.3845	0.3856	— .06	70	0.1659	0.1660	— .01	
28	60	0.6129	0.6126	+ .02	58	0.9371	0.9361	+ .06	
29	84	0.2345	0.2326	+ .10	35	0.3168	0.3154	+ .07	
33	19	0.3819	0.3838	— .10	100	0.1718	0.1716	+ .01	
		Schl.	Hays.	S—H		Schl.	Hays.	S—H	
15	56	0.6586	0.6598	—0.06	62	0.8946	0.8964	—0.11	
1	72	0.3909	0.3881	+ .15	47	0.1674	0.1626	+ .25	
5	85	0.6366	0.6354	+ .06	33	0.9176	0.9172	+ .03	
7	37	0.6385	0.6375	+ .05	81	0.9169	0.9132	+ .21	
23A	71	0.4271	0.4246	+ .13	48	0.1265	0.1219	+ .24	
31	40	0.8852	0.8874	— .13	78	0.6710	0.6731	— .11	
32	17	0.0584	0.0532	+ .28	102	0.5040	0.5041	— .01	
33	19	0.3802	0.3842	— .21	100	0.1728	0.1711	+ .09	
34	52	0.7965	0.7979	— .07	66	0.7548	0.7551	— .01	
35	90	0.0624	0.0638	— .07	29	0.4945	0.4920	+ .13	
36	67	0.5094	0.5024	+ .37	52	0.0486	0.0412	+ .39	
37	43	0.3850	0.3868	— .10	76	0.1650	0.1701	— .27	
38	60	0.3900	0.3834	+ .35	59	0.1671	0.1660	+ .06	
39	66	0.6850	0.6826	+ .13	52	0.8684	0.8680	+ .03	
40	21	0.7210	0.7229	— .10	97	0.8329	0.8329	.00	
43	63	0.5462	0.5432	+ .15	56	0.0040	0.0062	— .12	
44	82	0.9712	0.9688	+ .11	36	0.5834	0.5864	— .16	
45	32	0.1102	0.1076	+ .14	87	0.4465	0.4474	— .05	

III.

Instrumental Corrections.

The first step towards turning the foregoing measures into right ascensions and declinations will be to apply the following instrumental corrections, which are here considered in the order of their application.

1° *Division Errors of the Scale.*

Just before beginning the measurement of the Præsepe plates a thorough examination of the division errors of the scale was completed. The details of this investigation together with the determinations of other constants of the measuring machine are reserved for another publication from this observatory. It will suffice for present purposes to set down merely the final results.

Previous to the above investigation the scale had also been examined by the *Kaiserliche Normal Aichungs Kommission* at Berlin; the results of this determination were published in the "Annals of the New York Academy of Sciences," Vol. IX, page 206. The two determinations agree quite well, the largest difference for any line being 0."11, and usually the agreement is much closer. As the investigation at Columbia was made with the same microscope and under the same conditions in which the plates were measured, it was thought best to use only our own results, as given in Table IV. The coördinates of a star depend upon several divisions for one plate, and the same star usually comes opposite different divisions for different plates. It follows therefore that our final positions will be nearly independent of inaccuracies in the determination of the division errors; for example, the right ascension of Star 1 depends upon eighteen different lines of the scale and its declination depends upon thirteen. In Table IV the corrections are given in millimetres, and are always to be added to observed readings.

2° *Corrections for Runs and Screw Errors.*

The screw used in the measurements is of such a pitch that two complete turns of the micrometer head correspond to one space on the scale; the micrometer head is divided into one hundred equal parts and may therefore be read directly to half-microns

TABLE IV.—DIVISION ERRORS OF THE SCALE.

Line.	Correction in mm.	Line.	Correction in mm.	Line.	Correction in mm.
0	0.0000	44	+0.0029	88	+0.0025
1	+0.0011	45	+0.0020	89	+0.0026
2	-0.0007	46	+0.0021	90	+0.0018
3	-0.0001	47	+0.0006	91	+0.0027
4	+0.0002	48	+0.0013	92	+0.0025
5	-0.0004	49	+0.0015	93	+0.0026
6	+0.0001	50	+0.0007	94	+0.0026
7	-0.0013	51	+0.0018	95	+0.0026
8	-0.0008	52	+0.0030	96	+0.0019
9	-0.0009	53	+0.0024	97	+0.0008
10	-0.0001	54	+0.0032	98	+0.0014
11	+0.0014	55	+0.0028	99	+0.0004
12	+0.0006	56	+0.0030	100	+0.0019
13	+0.0014	57	+0.0034	101	+0.0004
14	+0.0016	58	+0.0038	102	-0.0005
15	+0.0015	59	+0.0034	103	+0.0002
16	+0.0009	60	+0.0034	104	+0.0014
17	+0.0008	61	+0.0038	105	+0.0009
18	+0.0012	62	+0.0044	106	+0.0012
19	+0.0012	63	+0.0058	107	+0.0002
20	+0.0007	64	+0.0050	108	+0.0012
21	+0.0015	65	+0.0053	109	0.0000
22	+0.0016	66	+0.0059	110	-0.0002
23	+0.0014	67	+0.0050	111	+0.0002
24	+0.0012	68	+0.0052	112	-0.0006
25	+0.0007	69	+0.0057	113	-0.0013
26	+0.0026	70	+0.0054	114	+0.0005
27	+0.0033	71	+0.0056	115	-0.0003
28	+0.0026	72	+0.0047	116	0.0000
29	+0.0020	73	+0.0046	117	+0.0020
30	+0.0035	74	+0.0047	118	+0.0022
31	+0.0030	75	+0.0037	119	+0.0026
32	+0.0022	76	+0.0052	120	+0.0021
33	+0.0024	77	+0.0045	121	+0.0014
34	+0.0013	78	+0.0041	122	+0.0024
35	+0.0032	79	+0.0040	123	+0.0015
36	+0.0024	80	+0.0033	124	+0.0024
37	+0.0025	81	+0.0043	125	+0.0012
38	+0.0016	82	+0.0029	126	+0.0018
39	+0.0025	83	+0.0050	127	+0.0014
40	+0.0033	84	+0.0034	128	+0.0005
41	+0.0020	85	+0.0021	129	+0.0015
42	+0.0025	86	+0.0014	130	0.0000
43	+0.0024	87	+0.0026		

and by estimation to twentieths of a micron. The details of the operation of observing runs were to set the micrometer head at about 5.^o and to read on the scale as follows:

Line 70, Line 65, Line 65, Line 70.

These two lines were selected because they have practically the same division errors, thus avoiding an extra correction, and also because they happen to have more accurately determined division errors than most other lines. The correction for runs need not be applied to the separate readings on the stars and on the scale but may be applied to the quantity $\frac{1}{2}m$ directly; for let us put

$2R$ = reading on Line 65, minus reading on Line 70, minus 10.^o0000.

Dividing by 2 we obtain R , the quantity given in Table II; we must then add to $\frac{1}{2}m$ the correction

$$- \left(\frac{1}{2}m\right) \frac{R}{5} \text{ millimetres.}$$

This correction may conveniently be combined with the correction for non-periodic errors of the screw or variations in its pitch; investigation showed that the following quantities must be added to observed readings of the micrometer head in order to reduce them to what they would have been had the screw been of uniform pitch, but of the same total length:

Reading of the Micrometer Head.	Correction in Millimetres.
5.0.....	0.
6.0.....	+ 0.0005
7.0.....	+ 0.0002
8.0.....	— 0.0003
9.0.....	— 0.0012
10.0.....	— 0.0017
11.0.....	— 0.0022
12.0.....	— 0.0021
13.0.....	— 0.0022
14.0.....	— 0.0014
15.0.....	0.

The screw is actually longer than ten turns, but as only this length was used in the determination of the error of runs the rest of the screw was not investigated. It will be observed that the corrections progress uniformly in the interval from 9.^o to 11.^o; advantage was taken of this fact to make the application of the corrections a simple matter. For if the micrometer head be set at 9.^o when the microscope is pointed at a star, then the reading

on the scale will lie between 9.^o and 11.^o because a whole space of the scale corresponds to two turns of the micrometer head; consequently the correction to the difference of the readings on scale and star will be proportionate to that difference, that is to $\frac{1}{2} m$; the correction to the latter is easily seen to be

$$-(\frac{1}{2} m) \times 0.0010 \text{ millimetres.}$$

Adding this to the correction for runs we obtain

$$-(\frac{1}{2} m) \left(0.00010 + \frac{R}{5} \right) \text{ millimetres.}$$

As an example let us correct the first observation given on the specimen sheet on page 199. The date being March 25, we get from Table II,

$$R = + 0.0045.$$

Consequently the correction for runs and screw errors is

$$-(\frac{1}{2} m) \times 0.0019 \text{ millimetres.}$$

A table may now be constructed with the argument $\frac{1}{2} m$ which applies to all observations taken on March 25.

$\frac{1}{2} m.$	Correction.
0.0.....	0.0000 millimetres.
0.1.....	— 0.0002
0.2.....	— 0.0004
0.3.....	— 0.0006
0.4.....	— 0.0008
0.5.....	— 0.0010
0.6.....	— 0.0011
0.7.....	— 0.0013
0.8.....	— 0.0015
0.9.....	— 0.0017
1.0.....	— 0.0019

For the star given on the specimen sheet the correction is — 0.0009. During the second half of each morning's observations when the micrometer head is set at 9.^r5 instead of at 9.^o it sometimes happens that the reading on the scale exceeds 11.^o, in which case the correction will not be exactly proportionate to $\frac{1}{2} m$; but the error committed by using the same table throughout will never reach 0."02, and in most cases is entirely negligible.

3°. Having applied the corrections given above we have now to change the measures into rectangular coördinates x and y , referred to the central star 15 as origin, one axis being parallel to the cylinder and the other at right angles to it. For this purpose we subtract the mean of all the readings on the central star for

any one day from the readings of all the other stars that were measured on that day. As we wish to have positive values of x for those stars which have greater right ascensions than the central star, we must subtract the reading " x direct" from the reading on Star 15; but we must subtract the reading on Star 15 from the reading " x reversed." Similarly to get positive values of y for those stars having greater declinations than the central star, we subtract the reading on Star 15 from each " y direct" and the contrary for " y reversed."

4° Rotation Corrections.

It was found very difficult to set the circle at exactly 90° plus the reading for the previous day. Even when this had been accomplished the circle-reading was sometimes found to have changed a little during the measurement of the stars. A correction is therefore necessary to reduce the rectangular coördinates to what they would have been had the readings of the circle for different days differed only by multiples of 90° . Let

Q = the number of seconds which occurs most often in the circle readings of a particular plate.

$Q - i$ = the number of seconds in the reading for any day.

Then we have,*

$$\begin{aligned} \text{Correction for } x &= -y \cdot i \sin 1.'' \\ \text{" " } y &= +x \cdot i \sin 1.'' \end{aligned}$$

For the present measurements these corrections are very small, never exceeding $0.''05$; they have however been applied throughout.

The values of Q adopted for the various plates are as follows:

Plate	I.....	$Q = 5''$
	II.....	0
	III.....	0
	IV.....	58
	V.....	7
	VII.....	0
	VIII.....	29
	IX.....	2

5° Scale-value corrections.

The scale being made of German silver has a greater coefficient of expansion than the glass plate, and hence it would appear that if the temperature changed during the measurement of a plate,

* "Permanence of the Rutherford Photographic Plates" by Harold Jacoby, *Annals of the N. Y. Acad. of Sciences*, Vol. IX, p. 267.

the coördinates would require a correction to reduce them to what they would have been had the temperature remained constant. Investigation shows, however, that such a correction is unnecessary by reason of its minuteness, at least within the limits of the range of temperature at which the present plates were measured. To ascertain the amount of the correction, two well defined specks, such as may be found in the film of any plate, were selected, one near either edge of the plate, and the distance between them was measured at various temperatures. On the morning of April 30, 1897, this distance was measured six times each by Mr. Kretz and myself with the following results :

Schlesinger :		Kretz :	
	104.1548 mm.		104.1505 mm.
	67		514
	36		505
	30		497
	50		540
	25		517
Mean,	104.1543		104.1513
Probable Error,	± 0.00042		± 0.00041

The temperature of the measuring room had been kept at $69.^{\circ}3$ during the measurement by means of artificial heat, the heating apparatus being at the other end of the room from that occupied by the measuring machine. The heat was now turned off and the plate allowed to assume the natural temperature of the atmosphere ; on the afternoon of the same day, three hours having elapsed since the first series was completed, the distance between the specks was again measured by the same observers as follows :

Schlesinger :		Kretz :	
	104.1550 mm.		104.1557 mm.
	70		44
	63		48
	87		60
	90		53
	70		75
Mean,	104.1572		104.1556
Probable Error,	± 0.00041		± 0.00031

The temperature for this series was $52.^{\circ}2$. Denoting by v the increase in the measured distance due to an increase of 1° in the temperature we have

Schlesinger :

$$v = -0.00017 \text{ mm.}$$

$$\pm 0.000034$$

Kretz :

$$v = -0.00025 \text{ mm.}$$

$$\pm 0.000030$$

These two values are not very accordant, but they agree sufficiently well for present purposes. As, however, some doubts were entertained as to whether the plates had been thoroughly cooled in the intervening three hours, and as to whether it would not be better to measure the distance between the specks at a season when artificial heat could be entirely dispensed with, the following third series of observations was undertaken by myself, several days elapsing between the various measurements. No artificial heat was allowed in the measuring room during the whole period.

May 20, 1897. Temperature = 73.°6.

104.1530 mm.

10

18

30

58

48

25

68

 Mean, 104.1536 $\pm$ 0.00047

May 22, 1897. Temperature = 69.°6.

104.1534 mm.

36

26

44

34

39

54

76

 Mean, 104.1543 $\pm$ 0.00036

May 29, 1897. Temperature = 71.°0.

104.1548 mm.

26

28

64

34

28

36

34

 Mean, 104.1537 $\pm$ 0.00031

July 27, 1897. Temperature = $82.^{\circ}\text{I}$.

104.1532 mm.

525

505

530

520

495

510

542

510

522

548

525

Mean, 104.1522 $\pm$ 0.00031

Denoting by v as before the increase in the distance due to an increase of 1° in the temperature, and by L the distance between the specks at the temperature $69.^{\circ}6$ we have the following four observation equations :

$$\begin{array}{rcll} L & - & 104.1543 = 0 & \text{weight 3} \\ L + 1.4 v & - & 104.1537 = 0 & \text{" 5} \\ L + 4.0 v & - & 104.1536 = 0 & \text{" 2} \\ L + 12.5 v & - & 104.1522 = 0 & \text{" 5} \end{array}$$

The weights are calculated from the probable errors given above, the probable error of an equation of weight unity being

$$\pm 0.00067$$

Solving by least squares we get

$$v = -0.00015 \pm 0.000032$$

Taking into account the two previous series, the mean by weight is

$$v = -0.00019 \text{ mm.}$$

which very probably does not differ from its true value by as much as 0.00005.

Now the largest coördinate in the Præsepe measures is less than forty millimetres, and the greatest deviation for any single day from the mean of the temperatures for the corresponding plate is less than 5° ; consequently the largest correction which it will ever be necessary to apply is

$$0.00037 \text{ mm.}$$

which corresponds to 0."02. As this is so small, even in the extreme case, no appreciable error will be committed by neglecting the correction altogether. We may indeed conclude that for the

scale under consideration and for the Rutherford plates, a correction for change in the scale-value will be unnecessary so long as the temperature does not vary more than 10° during the measurement of a single plate.

6° Projection Errors and Deviation of the Cylinder from Straightness.

In the present work corrections have been applied for neither of these; the first were discussed by Donner* and are very small in most cases; it is indeed very difficult to determine them with sufficient accuracy. Repsold has recently devised a new guiding way which is free from this source of error; the measuring machine used for the Præsepe plates had been furnished with such a guiding way in 1896. As regards the straightness of the cylinder, investigation showed that it had been admirably made†; the greatest error which we shall commit in assuming it to be straight is 0.004 ; the reversal of each plate and the insertion of different plates in different positions in the measuring machine will tend to eliminate even this small error.

Having now completed the consideration of all the instrumental corrections which it is necessary to apply, I shall conclude this subject by correcting the measurements of Star 7, Plate VIII, as given on page 199, or in Table III.

	March 25, x direct:	March 24, y direct:	March 18, x reversed:	March 22, y reversed:
Lines,	74.76	36	43.44	83
$\frac{1}{2}m$; First obs'v'r., ..	0.4646	— 0.0154	0.5938	0.5680
Second " ..	0.4639	— 0.0178	0.5896	0.5670
Mean,	0.4643	— 0.0166	0.5917	0.5675
Cor. for Runs, etc., —	9	0	11	11
Div. Correction, +	50	+ 24	+ 26	+ 50
Corrected $\frac{1}{2}m$,	0.4684	— 0.0142	0.5932	0.5714
Measurement,	75.4684	35.9858	44.0932	83.5714

Where two lines have been used the division correction is the mean of the division corrections for the separate lines; the final "measurement" is then obtained by adding the mean of the numbers of the lines to the corrected $\frac{1}{2}m$. The corresponding measurements for the central star 15 when similarly corrected are:

* "Détermination des Constantes nécessaires pour la Reduction des Clichés." *Acta Societatis Scientiarum Fennicae*, Vol. XXI.

† "Permanence of the Rutherford Plates" by Harold Jacoby; *Annals of the N. Y. Acad. of Sciences*, Vol. IX, page 210.

March 25, x direct :	March 24, y direct :	March 18, x reversed :	March 22, y reversed :
60.7692	55.0133	58.7883	64.5497
7695	0104	7887	5457
7731	0100	7870	5448
7712	0123	7871	5450
Mean, 60.7708	55.0115	58.7878	64.5463

Taking the differences between these and the corresponding measurements for Star 7, having regard to signs, we get :

Coördinates,	—14.6976	—19.0257	—14.6946	—19.0251
Rotation Cor.,	0	+ 4	+ 2	0
Final Coörd.,	—14.6976	—19.0253	—14.6944	—19.0251

These are the quantities given in Table V, which needs no further explanation. In comparing the direct with the reversed coördinates, it should be remembered that unity in the fourth decimal place corresponds to about 0."005 of arc of a great circle.

TABLE V.—CORRECTED COÖRDINATES. PLATE I.

Star.	<i>x</i>			<i>y</i>		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.5052	.5028	—34.5040	+15.6854	.6850	+15.6852
2	—33.5104	.5082	—33.5093	— 3.4200	.4188	— 3.4194
3	—32.3942	.3922	—32.3932	—18.0042	.0055	—18.0049
4	—30.6393	.6348	—30.6370	—35.0918	.0906	—35.0912
5	—21.6168	.6161	—21.6165	+28.9521	.9571	+28.9546
6	—17.4281	.4236	—17.4258	+14.2048	.2048	+14.2048
7	—14.6839	.6843	—14.6841	—19.0394	.0412	—19.0403
8	—10.5876	.5849	—10.5862	+14.9500	.9500	+14.9500
10	— 9.6888	.6886	— 9.6887	— 7.0204	.0201	— 7.0203
11	— 8.8264	.8270	— 8.8267	—12.7576	.7577	—12.7576
14	— 0.2020	.2022	— 0.2021	+20.9438	.9455	+20.9446
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.9159	.9173	+ 0.9166	—34.2424	.2429	—34.2426
17	+ 2.3272	.3293	+ 2.3283	—16.1534	.1504	—16.1519
18	+ 3.8697	.8712	+ 3.8705	—15.3037	.3022	—15.3030
20	+ 4.2144	.2182	+ 4.2163	—16.0818	.0826	—16.0822
22	+ 6.1437	.1546	+ 6.1541	+15.6744	.6760	+15.6752
23	+ 7.5258	.5276	+ 7.5267	+13.2000	.2000	+13.2000
23A	+ 8.5446	.5478	+ 8.5462	+14.7914	.7874	+14.7894
24	+ 9.6569	.6587	+ 9.6578	—16.6650	.6664	—16.6657
25	+10.2385	.2420	+10.2402	— 6.1255	.1240	— 6.1248
26	+10.3560	.3580	+10.3570	—29.1131	.1110	—29.1120
27	+10.6100	.6095	+10.6098	— 7.2644	.2646	— 7.2645
28	+10.8422	.8406	+10.8414	+ 3.9756	.9756	+ 3.9756
29	+11.4264	.4272	+11.4268	+27.6000	.5983	+27.5992
31	+12.0292	.0308	+12.0300	—15.7662	.7623	—15.7642
32	+13.9115	.9104	+13.9110	—39.5974	.5944	—39.5959
33	+15.5667	.5705	+15.5686	—37.2662	.2662	—37.2662
34	+16.2032	.2048	+16.2040	— 3.8424	.8368	— 3.8396
35	+18.1994	.1978	+18.1986	+33.4348	.4344	+33.4346
36	+19.6786	.6824	+19.6805	+10.8790	.8794	+10.8792
37	+19.8224	.8254	+19.8239	—13.2516	.2506	—13.2512
38	+23.4340	.4385	+23.4362	+ 3.7612	.7620	+ 3.7616
39	+24.1815	.1872	+24.1844	+10.0636	.0626	+10.0631
40	+26.0190	.0207	+26.0198	—34.9100	.9092	—34.9096
43	+33.9119	.9091	+33.9105	+ 6.9450	.9450	+ 6.9450
44	+34.4409	.4470	+34.4440	+26.3683	.3666	+26.3674

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE II.

Star.	x			y		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.5040	.5037	—34.5038	+15.6928	.6954	+15.6941
2	—33.5044	.5036	—33.5040	— 3.4064	.4061	— 3.4062
4	—30.6440	.6420	—30.6430	—35.0756	.0789	—35.0772
5	—21.6032	.6037	—21.6035	+28.9578	.9610	+28.9594
6	—17.4227	.4214	—17.4221	+14.2064	.2055	+14.2059
7	—14.6867	.6812	—14.6840	—19.0368	.0348	—19.0358
7A	—10.5894	.5874	—10.5884	— 3.7603	.7585	— 3.7594
8	—10.5806	.5799	—10.5803	+14.9550	.9538	+14.9544
10	— 9.6862	.6838	— 9.6850	— 7.0164	.0154	— 7.0159
11	— 8.8272	.8259	— 8.8266	—12.7520	.7530	—12.7525
14	— 0.2004	.2006	— 0.2005	+20.9444	.9442	+20.9443
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.9121	.9122	+ 0.9122	—34.2378	.2359	—34.2368
17	+ 2.3210	.3230	+ 2.3220	—16.1492	.1480	—16.1486
18	+ 3.8660	.8664	+ 3.8662	—15.3018	.3032	—15.3025
19	+ 3.8978	.8967	+ 3.8972	+25.8933	.8950	+25.8942
20	+ 4.2114	.2147	+ 4.2130	—16.0802	.0786	—16.0794
22	+ 6.1538	.1518	+ 6.1528	+13.1972	.1972	+13.1972
23A	+ 8.5476	.5505	+ 8.5490	+14.7833	.7834	+14.7834
24	+ 9.6502	.6500	+ 9.6501	—16.6680	.6658	—16.6669
25	+10.2337	.2350	+10.2344	— 6.1280	.1255	— 6.1268
26	+10.3454	.3492	+10.3473	—29.1082	.1090	—29.1086
27	+10.6000	.6032	+10.6016	— 7.2620	.2598	— 7.2609
28	+10.8382	.8390	+10.8386	+ 3.9732	.9757	+ 3.9744
29	+11.4256	.4253	+11.4254	+27.5944	.5926	+27.5935
31	+12.0266	.0287	+12.0276	—15.7620	.7641	—15.7630
32	+13.9074	.9049	+13.9061	—39.5946	.5894	—39.5920
33	+15.5584	.5616	+15.5600	—37.2621	.2607	—37.2614
34	+16.2032	.2031	+16.2031	— 3.8444	.8394	— 3.8419
35	+18.2036	.2038	+18.2037	+33.4284	.4314	+33.4299
36	+19.6828	.6814	+19.6821	+10.8745	.8752	+10.8749
37	+19.8174	.8190	+19.8182	—13.2498	.2495	—13.2496
38	+23.4373	.4378	+23.4375	+ 3.7584	.7606	+ 3.7595
39	+24.1793	.1830	+24.1812	+10.0571	.0604	+10.0587
40	+26.0170	.0153	+26.0162	—34.9106	.9091	—34.9098
43	+33.9056	.9042	+33.9049	+ 6.9342	.9360	+ 6.9351
44	+34.4474	.4480	+34.4477	+26.3596	.3632	+26.3614
45	+38.7409	.7417	+38.7413	—24.5064	.5066	—24.5065

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE III.

Star.	x			y		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.4942	.4942	—34.4942	+15.7075	.7076	+15.7076
2	—33.5073	.5094	—33.5084	— 3.3957	.3940	— 3.3948
3	—32.4044	.4030	—32.4037	—17.9840	.9844	—17.9842
4	—30.6587	.6538	—30.6562	—35.0679	.0660	—35.0670
5	—21.5902	.5939	—21.5920	+28.9642	.9662	+28.9652
6	—17.4130	.4134	—17.4132	+14.2124	.2157	+14.2141
7	—14.6915	.6974	—14.6954	—19.0312	.0342	—19.0327
8	—10.5682	.5685	—10.5684	+14.9584	.9602	+14.9593
10	— 9.6864	.6859	— 9.6862	— 7.0159	.0138	— 7.0148
11	— 8.8284	.8312	— 8.8298	—12.7523	.7510	—12.7517
14	— 0.1866	.1878	— 0.1872	+20.9390	.9420	+20.9405
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.8979	.8959	+ 0.8969	—34.2431	.2410	—34.2420
17	+ 2.3194	.3190	+ 2.3192	—16.1527	.1504	—16.1516
18	+ 3.8596	.8590	+ 3.8593	—15.3089	.3060	—15.3074
20	+ 4.2056	.2076	+ 4.2066	—16.0852	.0818	—16.0835
22	+ 6.1663	.1653	+ 6.1658	+15.6676	.6737	+15.6706
23	+ 7.5299	.5297	+ 7.5298	+13.1907	.1931	+13.1919
23A	+ 8.5566	.5554	+ 8.5560	+14.7819	.7820	+14.7820
24	+ 9.6504	.6490	+ 9.6497	—16.6740	.6688	—16.6714
25	+10.2314	.2295	+10.2305	— 6.1371	.1310	— 6.1340
26	+10.3361	.3369	+10.3365	—29.1205	.1154	—29.1180
27	+10.6036	.6032	+10.6034	— 7.2689	.2712	— 7.2700
28	+10.8441	.8452	+10.8447	+ 3.9637	.9659	+ 3.9648
29	+11.4440	.4462	+11.4451	+27.5865	.5912	+27.5888
31	+12.0134	.0165	+12.0150	—15.7675	.7700	—15.7688
32	+13.8892	.8889	+13.8891	—39.6008	.6048	—39.6028
33	+15.5390	.5422	+15.5406	—37.2750	.2731	—37.2740
34	+16.2035	.2034	+16.2034	— 3.8514	.8503	— 3.8508
35	+18.2207	.2234	+18.2221	+33.4219	.4217	+33.4218
36	+19.6910	.6868	+19.6889	+10.8652	.8620	+10.8636
37	+19.8112	.8124	+19.8118	—13.2630	.2656	—13.2643
39	+24.1897	.1879	+24.1888	+10.0472	.0462	+10.0467
40	+25.9978	.0012	+25.9995	—34.9216	.9212	—34.9214
43	+33.9138	.9114	+33.9126	+ 6.9199	.9170	+ 6.9184
44	+34.4648	.4650	+34.4649	+26.3452	.3448	+26.3450
45	+38.7302	.7281	+38.7292	—24.5248	.5266	—24.5257

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE IV.

Star.	<i>x</i>			<i>y</i>		
	Direct.	Rev'd.	Mean.	D.rect.	Rev'd.	Mean.
1	—34.5081	.5005	—34.5043	+15.6765	.6747	+15.6756
2	—33.5014	.4986	—33.5000	— 3.4249	.4234	— 3.4241
3	—32.3816	.3810	—32.3813	—18.0095	.0107	—18.0101
4	—30.6188	.6162	—30.6175	—35.0890	.0904	—35.0897
5	—21.6262	.6230	—21.6246	+28.9437	.9424	+28.9430
6	—17.4268	.4245	—17.4256	+14.1982	.2015	+14.1999
7	—14.6724	.6690	—14.6707	—19.0377	.0411	—19.0394
7A	—10.5836	.5818	—10.5827	— 3.7664	.7693	— 3.7678
8	—10.5906	.5902	—10.5904	+14.9470	.9494	+14.9482
10	— 9.6824	.6814	— 9.6819	— 7.0229	.0209	— 7.0219
11	— 8.8172	.8155	— 8.8164	—12.7601	.7583	—12.7592
14	— 0.2114	.2107	— 0.2110	+20.9386	.9403	+20.9394
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.9326	.9340	+ 0.9333	—34.2394	.2372	—34.2383
17	+ 2.3349	.3362	+ 2.3356	—16.1490	.1489	—16.1490
18	+ 3.8712	.8732	+ 3.8722	—15.3017	.3015	—15.3016
19	+ 3.8816	.8832	+ 3.8824	+25.8905	.8886	+25.8896
20	+ 4.2190	.2192	+ 4.2191	—16.0778	.0775	—16.0777
22	+ 6.1440	.1465	+ 6.1452	+15.6732	.6741	+15.6737
23	+ 7.5157	.5178	+ 7.5168	+13.1976	.1967	+13.1971
23A	+ 8.5374	.5399	+ 8.5387	+14.7866	.7864	+14.7865
24	+ 9.6622	.6640	+ 9.6631	—16.6652	.6606	—16.6629
25	+10.2376	.2372	+10.2374	— 6.1234	.1220	— 6.1227
26	+10.3627	.3652	+10.3639	—29.1068	.1060	—29.1064
27	+10.6071	.6077	+10.6074	— 7.2612	.2614	— 7.2613
28	+10.8355	.8374	+10.8364	+ 3.9759	.9771	+ 3.9765
29	+11.4120	.4129	+11.4124	+27.5930	.5960	+27.5945
31	+12.0288	.0305	+12.0297	—15.7623	.7610	—15.7616
32	+13.9279	.9289	+13.9284	—39.5870	.5883	—39.5876
33	+15.5832	.5771	+15.5802	—37.2559	.2551	—37.2555
34	+16.2028	.2030	+16.2029	— 3.8344	.8350	— 3.8347
35	+18.1752	.1798	+18.1775	+33.4338	.4318	+33.4328
36	+19.6722	.6738	+19.6730	+10.8808	.8800	+10.8804
37	+19.8242	.8250	+19.8246	—13.2450	.2459	—13.2454
38	+23.4288	.4324	+23.4306	+ 3.7647	.7636	+ 3.7642
39	+24.1717	.1728	+24.1722	+10.0655	.0655	+10.0655
40	+26.0288	.0312	+26.0300	—34.8968	.8938	—34.8953
43	+33.8955	.8986	+33.8970	+ 6.9446	.9455	+ 6.9450
44	+34.4188	.4232	+34.4210	+26.3714	.3697	+26.3706
45	+38.7495	.7503	+38.7499	—24.4909	.4900	—24.4904

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE V.

Star.	<i>x</i>			<i>y</i>		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.4912	.4900	—34.4906	+15.7164	.7130	+15.7147
2	—33.5065	.5062	—33.5064	— 3.3860	.3897	— 3.3878
3	—32.4083	.4072	—32.4077	—17.9751	.9772	—17.9762
4	—30.6656	.6644	—30.6650	—35.0600	.0620	—35.0610
5	—21.5824	.5810	—21.5817	+28.9692	.9680	+28.9686
6	—17.4077	.4090	—17.4084	+14.2193	.2187	+14.2190
7	—14.6985	.6980	—14.6982	—19.0254	.0321	—19.0288
8	—10.5690	.5682	—10.5686	+14.9581	.9598	+14.9589
10	— 9.6874	.6894	— 9.6884	— 7.0102	.0116	— 7.0109
11	— 8.8342	.8320	— 8.8331	—12.7484	.7500	—12.7492
14	— 0.1788	.1805	— 0.1797	+20.9386	.9374	+20.9380
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.8868	.8872	+ 0.8870	—34.2418	.2394	—34.2406
17	+ 2.3130	.3152	+ 2.3141	—16.1528	.1520	—16.1524
18	+ 3.8581	.8571	+ 3.8576	—15.3042	.3044	—15.3043
19	+ 3.9180	.9201	+ 3.9190	+25.8858	.8814	+25.8836
20	+ 4.2016	.2034	+ 4.2025	—16.0850	.0826	—16.0838
22	+ 6.1697	.1684	+ 6.1690	+15.6671	.6672	+15.6672
23	+ 7.5392	.5374	+ 7.5383	+13.1906	.1916	+13.1911
23A	+ 8.5584	.5599	+ 8.5592	+14.7780	.7756	+14.7768
24	+ 9.6428	.6431	+ 9.6429	—16.6726	.6745	—16.6736
25	+10.2291	.2308	+10.2299	— 6.1376	.1352	— 6.1364
26	+10.3236	.3257	+10.3246	—29.1198	.1210	—29.1204
27	+10.6022	.6037	+10.6030	— 7.2723	.2700	— 7.2711
28	+10.8461	.8451	+10.8456	+ 3.9626	.9622	+ 3.9624
29	+11.4492	.4498	+11.4495	+27.5812	.5812	+27.5812
31	+12.0096	.0100	+12.0098	—15.7728	.7723	—15.7726
32	+13.8710	.8778	+13.8744	—39.6013	.6038	—39.6025
34	+16.1957	.1977	+16.1967	— 3.8561	.8526	— 3.8543
35	+18.2316	.2267	+18.2292	+33.4117	.4098	+33.4108
37	+19.8083	.8070	+19.8077	—13.2664	.2697	—13.2680
39	+24.1869	.1882	+24.1876	+10.0364	.0376	+10.0370
40	+25.9806	.9837	+25.9822	—34.9250	.9260	—34.9255
43	+33.9056	.9072	+33.9064	+ 6.9073	.9062	+ 6.9068
44	+34.4653	.4661	+34.4657	+26.3334	.3336	+26.3335
45	+38.7196	.7170	+38.7183	—24.5324	.5328	—24.5326

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE VII.

Star.	<i>x</i>			<i>y</i>		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.4817	.4843	—34.4830	+15.7221	.7231	+15.7226
2	—33.5043	.5043	—33.5043	— 3.3759	.3763	— 3.3761
3	—32.4086	.4100	—32.4093	—17.9669	.9658	—17.9664
4	—30.6694	.6673	—30.6684	—35.0474	.0472	—35.0473
5	—21.5742	.5714	—21.5728	+28.9724	.9737	+28.9730
6	—17.4069	.4056	—17.4062	+14.2230	.2216	+14.2223
7	—14.7014	.7022	—14.7018	—19.0236	.0206	—19.0221
7A	—10.5922	.5894	—10.5908	— 3.7542	.7547	— 3.7545
8	—10.5673	.5687	—10.5680	+14.9603	.9586	+14.9595
10	— 9.6920	.6904	— 9.6912	— 7.0050	.0060	— 7.0055
11	— 8.8388	.8402	— 8.8395	—12.7439	.7446	—12.7443
14	— 0.1793	.1816	— 0.1805	+20.9396	.9388	+20.9392
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.8806	.8805	+ 0.8806	—34.2354	.2361	—34.2359
17	+ 2.3125	.3090	+ 2.3108	—16.1506	.1493	—16.1500
18	+ 3.8524	.8514	+ 3.8519	—15.3052	.3057	—15.3054
19	+ 3.9198	.9222	+ 3.9210	+25.8832	.8854	+25.8843
20	+ 4.1979	.1962	+ 4.1971	—16.0844	.0854	—16.0849
21	+ 4.7178	.7213	+ 4.7196	+24.3594	.3554	+24.3574
22	+ 6.1656	.1624	+ 6.1640	+15.6664	.6633	+15.6648
23	+ 7.5330	.5340	+ 7.5335	+13.1890	.1872	+13.1881
23A	+ 8.5558	.5590	+ 8.5574	+14.7731	.7745	+14.7738
24	+ 9.6320	.6332	+ 9.6326	—16.6755	.6736	—16.6745
25	+10.2276	.2252	+10.2264	— 6.1370	.1348	— 6.1359
26	+10.3185	.3214	+10.3199	—29.1174	.1180	—29.1177
27	+10.5986	.5979	+10.5982	— 7.2726	.2710	— 7.2718
28	+10.8415	.8406	+10.8410	+ 3.9608	.9590	+ 3.9599
29	+11.4500	.4474	+11.4487	+27.5814	.5797	+27.5806
31	+12.0062	.0064	+12.0063	—15.7710	.7727	—15.7718
32	+13.8671	.8658	+13.8664	—39.6006	.6022	—39.6014
33	+15.5207	.5220	+15.5214	—37.2730	.2721	—37.2726
34	+16.1972	.1980	+16.1976	— 3.8560	.8536	— 3.8548
35	+18.2277	.2290	+18.2284	+33.4081	.4088	+33.4084
36	+19.6808	.6854	+19.6831	+10.8540	.8588	+10.8564
37	+19.8020	.8021	+19.8020	—13.2730	.2723	—13.2726
38	+23.4334	.4379	+23.4356	+ 3.7330	.7316	+ 3.7323
39	+24.1835	.1831	+24.1833	+10.0310	.0340	+10.0325
40	+25.9716	.9742	+25.9729	—34.9286	.9282	—34.9284
41	+30.0359	.0402	+30.0381	— 7.5610	.5599	— 7.5605
42	+32.0193	.0197	+32.0195	— 7.6587	.6003	— 7.6595
43	+33.9009	.9016	+33.9012	+ 6.8994	.9014	+ 6.9004
44	+34.4624	.4636	+34.4630	+26.3208	.3246	+26.3227
45	+38.7094	.7085	+38.7090	—24.5388	.5361	—24.5374

TABLE V.—(Continued.) CORRECTED COÖRDINATES. PLATE VIII.

Star.	α			δ		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean.
1	—34.4857	.4866	—34.4862	+15.7100	.7124	+15.7112
2	—33.5032	.5024	—33.5028	— 3.3848	.3846	— 3.3847
3	—32.4060	.4011	—32.4036	—17.9734	.9738	—17.9736
4	—30.6600	.6560	—30.6580	—35.0517	.0524	—35.0520
5	—21.5811	.5808	—21.5809	+28.9670	.9680	+28.9675
6	—17.4127	.4101	—17.4114	+14.2171	.2192	+14.2182
7	—14.6976	.6944	—14.6960	—19.0253	.0251	—19.0252
7A	—10.5962	.5969	—10.5966	— 3.7603	.7626	— 3.7615
8	—10.5749	.5718	—10.5733	+14.9556	.9570	+14.9563
10	— 9.6885	.6880	— 9.6882	— 7.0098	.0096	— 7.0097
11	— 8.8362	.8352	— 8.8357	—12.7488	.7488	—12.7488
14	— 0.1844	.1852	— 0.1848	+20.9374	.9392	+20.9383
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.8857	.8885	+ 0.8871	—34.2371	.2353	—34.2362
17	+ 2.3126	.3134	+ 2.3130	—16.1488	.1500	—16.1494
18	+ 3.8562	.8560	+ 3.8561	—15.3028	.3060	—15.3044
19	+ 3.9104	.9097	+ 3.9100	+25.8812	.8832	+25.8822
20	+ 4.2000	.2031	+ 4.2016	—16.0818	.0820	—16.0819
22	+ 6.1610	.1636	+ 6.1623	+15.6656	.6687	+15.6672
23	+ 7.5284	.5337	+ 7.5310	+13.1874	.1886	+13.1880
23A	+ 8.5506	.5521	+ 8.5513	+14.7729	.7760	+14.7744
24	+ 9.6418	.6438	+ 9.6428	—16.6723	.6726	—16.6724
25	+10.2268	.2256	+10.2262	— 6.1372	.1347	— 6.1360
26	+10.3278	.3292	+10.3285	—29.1152	.1163	—29.1158
27	+10.5974	.5974	+10.5974	— 7.2728	.2689	— 7.2708
28	+10.8416	.8408	+10.8412	+ 3.9608	.9648	+ 3.9628
29	+11.4424	.4426	+11.4425	+27.5784	.5800	+27.5792
31	+12.0110	.0137	+12.0124	—15.7707	.7748	—15.7727
32	+13.8756	.8780	+13.8768	—39.6012	.6014	—39.6013
33	+15.5298	.5320	+15.5309	—37.2751	.2712	—37.2732
34	+16.1969	.1962	+16.1966	— 3.8543	.8544	— 3.8544
35	+18.2171	.2166	+18.2168	+33.4060	.4086	+33.4073
36	+19.6751	.6778	+19.6764	+10.8545	.8562	+10.8554
37	+19.8014	.8057	+19.8035	—13.2662	.2670	—13.2666
38	+23.4306	.4318	+23.4312	+ 3.7362	.7382	+ 3.7372
39	+24.1790	.1761	+24.1776	+10.0346	.0351	+10.0348
40	+25.9807	.9800	+25.9804	—34.9247	.9232	—34.9240
41	+30.0339	.0342	+30.0340	— 7.5534	.5538	— 7.5536
42	+32.0097	.0144	+32.0120	— 7.6577	.6576	— 7.6576
43	+33.8966	.8960	+33.8963	+ 6.9064	.9093	+ 6.9079
44	+34.4516	.4494	+34.4505	+26.3255	.3264	+26.3259
45	+38.7120	.7129	+38.7124	—24.5322	.5326	—24.5324

TABLE V.—(Concluded.) CORRECTED COÖRDINATES. PLATE IX.

Star.	x			y		
	Direct.	Rev'd.	Mean.	Direct.	Rev'd.	Mean
1	—34.4800	.4842	—34.4821	+15.7316	.7334	+15.7325
2	—33.5050	.5025	—33.5037	— 3.3680	.3664	— 3.3672
3	—32.4120	.4079	—32.4100	—17.9574	.9570	—17.9572
4	—30.6766	.6746	—30.6756	—35.0392	.0384	—35.0388
5	—21.5667	.5659	—21.5663	+28.9755	.9802	+28.9778
6	—17.4034	.4030	—17.4032	+14.2269	.2304	+14.2286
7	—14.7092	.7056	—14.7074	—19.0220	.0194	—19.0208
8	—10.5617	.5612	—10.5614	+14.9613	.9682	+14.9648
10	— 9.6866	.6860	— 9.6863	— 7.0064	.0018	— 7.0041
11	— 8.8444	.8424	— 8.8434	—12.7406	.7372	—12.7389
14	— 0.1718	.1718	— 0.1718	+20.9388	.9407	+20.9397
15	0.0000	.0000	0.0000	0.0000	.0000	0.0000
16	+ 0.8652	.8684	+ 0.8668	—34.2368	.2330	—34.2349
17	+ 2.3019	.3048	+ 2.3033	—16.1486	.1504	—16.1495
18	+ 3.8459	.8481	+ 3.8470	—15.3053	.3054	—15.3054
20	+ 4.1952	.1981	+ 4.1966	—16.0804	.0809	—16.0807
22	+ 6.1756	.1759	+ 6.1758	+15.6626	.6655	+15.6640
23	+ 7.5408	.5429	+ 7.5418	+13.1893	.1902	+13.1898
23A	+ 8.5656	.5620	+ 8.5638	+14.7698	.7735	+14.7716
24	+ 9.6386	.6369	+ 9.6378	—16.6742	.6750	—16.6746
25	+10.2299	.2300	+10.2300	— 6.1412	.1352	— 6.1382
26	+10.3108	.3114	+10.3111	—29.1168	.1175	—29.1171
27	+10.5979	.5991	+10.5985	— 7.2740	.2707	— 7.2724
28	+10.8479	.8466	+10.8473	+ 3.9552	.9610	+ 3.9581
29	+11.4602	.4605	+11.4603	+27.5766	.5815	+27.5790
31	+12.0046	.0050	+12.0048	—15.7726	.7764	—15.7745
32	+13.8562	.8568	+13.8565	—39.6046	.6040	—39.6043
33	+15.5071	.5086	+15.5079	—37.2773	.2739	—37.2756
34	+16.1936	.1964	+16.1950	— 3.8618	.8610	— 3.8614
35	+18.2386	.2367	+18.2376	+33.4038	.4042	+33.4040
36	+19.6886	.6863	+19.6875	+10.8493	.8510	+10.8502
37	+19.7985	.8004	+19.7994	—13.2732	.2738	—13.2735
38	+23.4416	.4410	+23.4413	+ 3.7287	.7290	+ 3.7289
39	+24.1886	.1881	+24.1884	+10.0280	.0287	+10.0284
40	+25.9634	.9639	+25.9637	—34.9383	.9338	—34.9360
43	+33.9029	.9006	+33.9018	+ 6.8892	.8907	+ 6.8899
44	+34.4765	.4747	+34.4756	+26.3112	.3123	+26.3117
45	+38.7025	.7012	+38.7018	—24.5497	.5502	—24.5499

IV.

Method of Reduction.

The measured coördinates having been cleared of instrumental errors, it remains to convert them into right ascensions and declinations. For this purpose the following constants must be known for each plate:

1°. The right ascension of the centre of the plate, in this case the central star 15.

2°. The declination of the same star.

3°. The scale-value, or the number of seconds corresponding to one space on the scale.

4°. The orientation correction, or the angle through which we must rotate the coördinate axes in order that they may point respectively in the directions of a parallel of declination and a circle of declination through the central star.

The first plan that suggested itself for determining these constants was to employ the two existing heliometer researches upon the group. In 1856 to 1858 Professor Winnecke of Bonn measured the position angles and distances of forty-four stars from the central star 15; and in 1889 to 1892 Professor Schur of Göttingen triangulated thirty-eight stars and derived the places of seven more by measuring the position angles and distances from stars in the triangulation. The results of both researches were published in one volume by Professor Schur, part IV of the "Astronomische Mittheilungen von der K. Sternwarte zu Göttingen." The stars whose positions are there given include almost all those appearing on the photographs, and consequently very accurate values of the four constants could be obtained by comparing the measured coördinates of a large number of stars with their heliometer places. Another plan is to determine the constants by comparison with meridian observations. While this is not as accurate as the preceding, it has the advantage of being independent of the heliometer results, thus rendering a comparison with the latter more instructive.

The course that I have actually pursued is this: the constants were first determined for each plate separately by comparing the

coördinates of some of the stars with meridian observations. For a star which appears on all the plates we have thus eight determinations of its right ascension, and eight of its declination, and a catalogue of the group may now be formed by taking the means. Finally a least square solution was made to determine how much the constants would have to be changed on the average, so as to secure the best possible agreement with the heliometer places. It is evident that if we now apply to our catalogue positions the corrections which result from these average changes in the constants we shall obtain the same results as though we had used the heliometer places to determine the constants for the separate plates, and had then taken the means.

The meridian observations used are those quoted by Professor Schur in the work mentioned above; they were used by him to help fix the place of his triangulation in the sky. Five stars were observed, the central star and four others distributed symmetrically over the plate; they are admirably fitted for our purpose, being at sufficiently large distances from the central star to insure accurate determinations of the scale-value and orientation, and yet not so distant as to have their photographic images much distorted. Their magnitudes are such that they appear on all of the plates with good images. The stars were observed both at Berlin and Göttingen;* the positions obtained at the former observatory and reduced to the A. G. catalogue system are as follows:

Star.	Epoch.	Equinox of 1890.0.	
4	1890.26	8 ^h 31 ^m 28. ^s 444,	+19°39'00".65
5	.51	32 02. 359	20 35 28 .43
15	.26	33 23. 463	20 09 55 .38
40	.51	35 00. 573	19 39 04 .35
44	.26	35 33. 417	20 33 03 .28

Each star was observed four times, and the probable error of a single observation is given as

$$\begin{aligned} &\pm 0.^{\circ}012 \text{ in right ascension,} \\ &\pm 0.''25 \text{ in declination.} \end{aligned}$$

At Göttingen each star was observed six times with these results:

* "Astronomische Mittheilungen von der K. Sternwarte zu Göttingen," Part IV, pages 139 et seq.

Star.	Epoch.	Equinox of 1890.0.			
4	1891.52	8 ^h 31 ^m	28.425,	+19°38'59".83	
5	.37	32	02.388	20 35 27 .75	
15	.55	33	23.462	20 09 55 .52	
40	.89	35	00.607	19 39 04 .03	
44	.89	35	33.447	20 33 03 .40	

Giving these observations the weight $\frac{1}{3}$, and those at Berlin the weight unity, as was done by Professor Schur, we get finally :

Star.	Epoch.	Equinox of 1890.0.			
4	1890.58	8 ^h 31 ^m	28.440,	+19 39'00".45	
5	.72	32	02.366	20 35 28 .26	
15	.58	33	23.463	20 09 55 .42	
40	.86	35	00.582	19 39 04 .27	
44	.67	35	33.425	20 33 03 .31	

As the epochs of these observations are from thirteen to twenty years later than the dates of our plates, it is necessary to apply proper motions, for which the following values have been adopted :

4	—0.°0054	+0''.007
5	—0. 0005	+0 .038
15	—0. 0049	+0 .017
40	—0. 0040	+0 .011
44	—0. 0044	+0 .015

These are the values given by Professor Schur in his catalogue of the group, but they are not derived directly from a comparison of his places with those of Winnecke. Systematic corrections have been added to make the proper motion of the group as a whole conform with observations by Bradley and by Tobias Mayer; these corrections are :

$$-0.°0003 \qquad +0''.039.$$

The necessity for such a large correction in declination is accounted for by Professor Schur, by assuming that either in the Bonn or in the Göttingen observations, or perhaps in both, the declination of the group as a whole was incorrectly determined; we shall have occasion to refer to this circumstance later.

The plates were taken at practically only two dates, 1870.3 and 1877.3; applying the corresponding proper motions to the meridian observations and reducing them to the equinox of 1875.0, we get :

For Plates I, II, III and IV,

	Δa	$\Delta \delta$
4	-1723".34	-1856".74
5	-1220.23	+1531.02
15	0	0
40	+1459.38	-1849.36
44	+1948.13	+1390.16
	a_0	δ_0
15	128°07'55".26	+20°13'01".26

For Plates V, VII, VIII and IX,

	Δa	$\Delta \delta$
4	-1723".40	-1856".81
5	-1219.75	+1531.17
15	0	0
40	+1459.48	-1849.40
44	+1948.19	+1390.15
	a_0	δ_0
15	128°07'51".75	+20°13'01".38

Δa and $\Delta \delta$ are obtained by subtracting the right ascension and declination of Star 15 from those of the five comparison stars.

The numerical work in the problem before us, namely, to determine the constants of the plates, will be greatly decreased by first assuming an approximate scale-value and then determining how much this is in error. Such an approximate value is furnished by the reduction of Rutherford's photographs of the Pleiades where it was found that

$$1 \text{ millimetre} = 52.''87$$

Let us suppose that this scale-value has been applied to the measured coördinates x and y of each of the comparison stars, giving X and Y ; then the quantities $X \sec \delta_0$ and Y will be nearly equal to the corresponding Δa and $\Delta \delta$ respectively. The causes of difference are the following:

a. Transformation Corrections (see below), *Refraction, Precession, etc.*

b. Orientation, use of incorrect scale-value, etc.

c. Errors of observation, both in the measured coördinates and in the meridian places.

Let us first consider the causes of difference under *a*; we shall then determine the orientation, true scale-value etc., by comparing the corrected coördinates with the corresponding values of Δa

and $\Delta\delta$, eliminating the errors of observation as far as possible by means of a least-square solution.

I. TRANSFORMATION CORRECTIONS.

An astronomical photograph may be regarded as a central projection of a portion of the celestial sphere upon a tangent plane. The point on the plate which corresponds to the point of tangency is the foot of the perpendicular let fall from the optical centre of the object glass upon the plane of the plate. The rigorous relations between the rectilinear coördinates referred to this point as origin, and the right ascension and declination of a star, were given in simple form by Professor Turner in Vol. XVI, of "Observatory," page 374. Previous to this, however, Ball and Rambaut gave these relations in the form of series in their paper "On the Relative Positions of 223 stars in γ Persei," Transactions of the Royal Irish Academy, Vol. XXX, page 247. In our notation these formulas would be

$$\begin{aligned}\Delta\alpha - X \sec \delta_0 &= (X \sec \delta_0) Y \tan \delta_0 - \frac{1}{3} (X \sec \delta_0)^3 + (X \sec \delta_0) Y \tan^2 \delta_0 \\ \Delta\delta - Y &= -\frac{1}{3} (X \sec \delta_0)^2 \sin 2\delta_0 - \frac{1}{3} Y^3 - \frac{1}{2} (X \sec \delta_0)^2 Y\end{aligned}$$

The elegance of these formulas lies in the fact that the coefficients of the powers and products of X and Y , are functions of δ_0 only, and are therefore constant for a plate, or indeed for an entire zone. For most plates these series are sufficiently accurate, but when the declination or the measured coördinates are large they fail; in such cases we do not need to resort to the rigorous formulas but we have merely to extend the series to higher terms, as was done by Professor Jacoby in a review of a paper by Professor Donner, in the *Vierteljahrschrift* for 1895, page 114. In the same place, formulas are also given in which $\Delta\alpha$ and $\Delta\delta$ appear in the second members, instead of X and Y as above. Omitting terms of higher degree than the third, which is permissible for the *Præsepe* plates, these formulas may be written

$$\begin{aligned}\Delta\alpha - X \sec \delta_0 &= \Delta\alpha \cdot \Delta\delta \cdot \tan \delta_0 - \frac{1}{3} \Delta\alpha^3 \left(1 - \frac{2}{3} \sin^2 \delta_0\right) \\ \Delta\delta - Y &= -\frac{1}{3} \Delta\alpha^2 \cdot \sin 2\delta_0 - \frac{1}{2} \Delta\alpha^2 \cdot \Delta\delta \cdot \cos 2\delta_0 - \frac{1}{3} \Delta\delta^3\end{aligned}$$

The use of these formulas presupposes a knowledge of the approximate values of $\Delta\alpha$ and $\Delta\delta$ for each star. They possess two points of advantage over the inverse forms: first, there is one term less in the expression for $\Delta\alpha - X \sec \delta_0$; and second, they give slightly more accurate values for the corrections as they do

not involve errors of orientation, scale-value, etc., such as would be incurred through the use of the measured X and Y in the second members. I have therefore used the latter expressions for computing the transformation corrections given in Table VI, employing the heliometer values of $\Delta\alpha$ and $\Delta\delta$ given by Professor Schur. It will be remembered that Rutherford was careful to make the central star 15 coincide with the foot of the perpendicular let fall from the optical centre of the object glass upon the plane of the plate. As our measured coördinates are referred to this star as origin, Table VI applies equally to all the plates.

TABLE VI.—TRANSFORMATION CORRECTIONS.

Star.	$\Delta\alpha - X \sec \delta_0$	$\Delta\delta - Y$	Star.	$\Delta\alpha - X \sec \delta_0$	$\Delta\delta - Y$
1	—2.83	—3.01	24	—0.85	—0.22
2	+0.66	—2.80	25	—0.34	—0.26
3	+3.15	—2.58	26	—1.60	—0.23
4	+5.75	—2.23	27	—0.41	—0.28
5	—3.33	—1.22	28	+0.23	—0.29
6	—1.31	—0.77	29	+1.68	—0.36
7	+1.49	—0.52	31	—1.01	—0.35
7A	+0.21	—0.27	32	—2.92	—0.39
8	—0.84	—0.29	33	—3.08	—0.53
10	+0.36	—0.23	34	—0.34	—0.65
11	+0.60	—0.19	35	+3.24	—0.89
14	—0.02	—0.01	36	+1.13	—0.98
15	0.00	0.00	37	—1.41	—0.97
16	—0.16	+0.05	38	+0.45	—1.38
17	—0.20	—0.01	39	+1.28	—1.47
18	—0.31	—0.03	40	—4.84	—1.59
19	+0.54	—0.06	41	—1.24	—2.24
20	—0.36	—0.04	42	—1.35	—2.54
22	+0.51	—0.10	43	+1.19	—2.89
23	+0.53	—0.15	44	+4.79	—3.05
23A	+0.67	—0.18	45	—5.12	—3.66

II. CORRECTIONS FOR REFRACTION.

Formulas for clearing rectangular coördinates of the effects of refraction were first published by Dr. Rambaut in the "*Astronomische Nachrichten*," No. 3125. Professor Turner has shown how these may be much simplified by employing the coördinates of the zenith as projected upon the plate.* His formulas, however, will not serve in the transformation of rectangular coördinates into right ascensions and declinations unless we apply an

* Monthly Notices, R.A.S., November, 1893.

extra correction for orientation. (See note at the end of this paper.) The formulas that I have preferred to use are those given by Professor Jacoby;* while these are not quite so simple as those of Professor Turner, they take into account the orientation correction mentioned. Let

φ = the latitude, $+40^{\circ}44'$ in this case.

$\theta - \alpha_0$ = the hour angle of the centre of the plate, from Table I.

δ_0 = the declination of the centre, $+20^{\circ}13'$

β = the constant of refraction, computed in the usual way and then multiplied by $\frac{6}{5}$ to allow for the increased refrangibility of photographic rays.†

Now let us compute :

$$\tan N = \cos (\theta - \alpha_0) \cot \varphi$$

$$G = \cot (\delta_0 + N)$$

$$H = \tan (\theta - \alpha_0) \sin N \operatorname{cosec} (\delta_0 + N)$$

$$M_x = \beta (1 + H^2)$$

$$N_x = \beta (G - \tan \delta_0) H \sec \delta_0$$

$$M_y = \beta (G + \tan \delta_0) H \cos \delta_0$$

$$N_y = \beta (1 + G^2)$$

Then the corrections for refraction take the form :

$$\text{Correction for } X \sec \delta_0 = M_x \cdot X \sec \delta_0 + N_x \cdot Y$$

$$\text{“ “ “ “ } Y = M_y \cdot X \sec \delta_0 + N_y \cdot Y$$

The coefficients of $X \sec \delta_0$ and of Y in the second members are constant for an entire plate. We may then construct Table VII, in which the number of the plate is the argument.

TABLE VII.—REFRACTION COEFFICIENTS.

Plate.	M_x	N_x	M_y	N_y
I.	0.000356	0.000017	0.000114	0.000349
II.	0.000423	0.000042	0.000174	0.000375
III.	0.000404	0.000031	0.000153	0.000373
IV.	0.000523	0.000086	0.000255	0.000424
V.	0.000357	0.000015	0.000110	0.000354
VII.	0.000423	0.000042	0.000174	0.000375
VIII.	0.000491	0.000074	0.000233	0.000404
IX.	0.000377	0.000023	0.000131	0.000360

All the coefficients are positive.

* *Astronomical Journal*, No. 387.

† *Bulletin du Comité Permanent*, I, 464; and Scheiner and Rambaut, *Astron. Nach.* 3255.

III. PRECESSION AND NUTATION.

These merely change the position of the axes to which the group is referred; it follows therefore that their *differential* effect upon X and Y is simply to rotate the coördinate axes through a small angle. When we determine the constants of a plate by comparing the measures of some of the stars with their true positions, it is evident that we need apply no corrections for precession and nutation to the measures of the comparison stars. Thus if we employ the places of the latter as referred to the equinox of 1875.0, then the value for the orientation which we shall get will include the necessary correction for precession and nutation. On the other hand, if we correct the places of the comparison stars for precession and nutation, then it would not only be necessary to apply the resulting orientation correction to the *other* stars, but we should also have to apply to them additional corrections for precession and nutation.

IV. ABERRATION.

It was shown by Bessel* that aberration changes the position angles around a point equally, and changes the distances by a constant factor, no matter in what direction the distance is measured. Consequently, as in the case of precession and nutation, we need apply no correction for aberration to the measures of the comparison stars, since the resulting orientation and scale-value corrections will be appropriately modified to include its whole effect.

Thus we see that the coördinates of our five comparison stars need be corrected only for transformation and refraction. We must bear in mind, however, that the orientation and the scale-value which we shall then obtain are not the true values of these constants: the former must be corrected for precession, nutation and aberration, and the latter simply for aberration.

We are now ready to find the constants of the plates. Let p = the correction to the scale-value, so that the true scale-value is $52''.87 (1 + p)$.

r = the orientation correction, or the small angle through which the axes are to be rotated in the direction of decreasing position angles.

* "Astronomische Untersuchungen," Vol. I, page 207.

k = the number of seconds of arc of a great circle through which the axes are to be translated in the direction of decreasing right ascensions.

c = the number of seconds of arc of a great circle through which the axes are to be translated in the direction of decreasing declinations.

The corrections to the rectangular coördinates arising from p are then :

$$\begin{array}{l} \text{For } X, \quad + p \cdot X \\ \text{" } Y, \quad + p \cdot Y \end{array}$$

On account of the orientation corrections, remembering that r is small, we have the corrections :

$$\begin{array}{l} \text{For } X, \quad + r \cdot Y \\ \text{" } Y, \quad - r \cdot X \end{array}$$

Finally, k and c give the corrections :

$$\begin{array}{l} \text{For } X, \quad + k \\ \text{" } Y, \quad + c \end{array}$$

Combining all these corrections, we have :

$$\begin{array}{l} \text{For } X, \quad + p \cdot X + r \cdot Y + k \\ \text{" } Y, \quad + p \cdot Y - r \cdot X + c \end{array}$$

Let us now compute n_x and n_y for each comparison star, from the following equations :

$n_x \sec \delta_0 = X \sec \delta_0$ *plus* corrections for transformation and refraction, *minus* $\Delta\alpha$.

$n_y = Y$ *plus* corrections for transformation and refraction, *minus* $\Delta\delta$.

Then for each comparison star we have two equations of the following form from which to determine p , r , k and c :

$$\begin{array}{l} pX + rY + k + n_x = 0 \\ pY - rX + c + n_y = 0 \end{array}$$

Owing to the way in which the coefficients of the unknowns are repeated in these equations we do not need to make the least square solution in the usual manner, but as Professor Jacoby has pointed out,* we may find the unknowns very simply. Thus, let ν = the number of comparison stars, and let us denote by square brackets the sum of ν quantities.

* Monthly Notices of the Royal Astronomical Society, May, 1896.

Put

$$A = [XX] + [YY] - \frac{1}{\nu} ([X]^2 + [Y]^2)$$

$$C = [X \cdot n_x] + [Y \cdot n_y] - \frac{1}{\nu} ([X][n_x] + [Y][n_y])$$

$$E = [Y \cdot n_x] - [X \cdot n_y] - \frac{1}{\nu} ([Y][n_x] - [X][n_y])$$

Then we have :

$$p = -\frac{C}{A}, \text{ with the weight } A.$$

$$r = -\frac{E}{A}, \quad \text{“} \quad \text{“} \quad \text{“} \quad A.$$

$$k = -\frac{1}{\nu} ([X]p + [Y]r + [n_x]), \text{ with the weight, } \nu = \frac{[X]^2 + [Y]^2}{[XX] + [YY]}$$

$$e = -\frac{1}{\nu} ([Y]p - [X]r + [n_y]), \quad \text{“} \quad \text{“} \quad \text{“} \quad \nu = \frac{[X]^2 + [Y]^2}{[XX] + [YY]}$$

The following will be found a convenient check on the computations: the sum of the residuals for the right ascension equations is equal to zero, and similarly for the declination equations.*

The above method of solution is rendered still simpler in the present case, as we are going to use the same comparison stars for all the plates. Hence all the terms in the expressions for A , C and E are constant except those which involve n_x or n_y . Thus, selecting the coördinates of the comparison stars from any plate in Table V, and multiplying them by 52.87 we have, with sufficient accuracy :

	X	Y
4	-1620''	-1850''
5	-1140	+1530
15	0	0
40	+1370	-1850
44	+1820	+1390

Consequently, for all the plates,

* This is indeed a general check for any set of observation equations in which one of the unknowns enters with a constant coefficient; if this unknown is missing from some of the equations, then the sum of the residuals for those equations in which it does appear is equal to zero. The theorem may be easily modified to include the case of unequal weights.

$$A = 20,080,000$$

$$C = [X \cdot n_x] + [Y \cdot n_y] - 86 [n_x] + 156 [n_y]$$

$$E = [Y \cdot n_x] - [X \cdot n_y] + 156 [n_x] - 86 [n_y]$$

$$p = - \frac{C}{20,080,000}; \text{ weight, } 20,080,000$$

$$r = - \frac{E}{20,080,000}; \quad " \quad 20,080,000$$

$$k = - 86 p + 156 r - 0.20 [n_x]; \text{ weight, } 4.96$$

$$c = + 156 p + 86 r - 0.20 [n_y]; \quad " \quad 4.96$$

It now remains to show how the right ascensions and declinations of all the stars may be computed. The constants of the plate give rise to the corrections:

$$\begin{array}{l} \text{For } X \sec \delta_0, \quad + p \cdot X \sec \delta_0 + r \sec \delta_0 \cdot Y + k \sec \delta_0 \\ \quad " \quad Y \quad , \quad + p \cdot Y \quad \quad - r \cdot X \quad \quad + c \end{array}$$

The corrections for refraction are:

$$\begin{array}{l} \text{For } X \sec \delta_0, \quad + M_x \cdot X \sec \delta_0 + N_x \cdot Y \\ \quad " \quad Y \quad , \quad + M_y \cdot X \sec \delta_0 + N_y \cdot Y \end{array}$$

We have still to add corrections for transformation, which vary from star to star, but are the same for different plates. Now let us define a_1 and δ_1 as the *projected* right ascension and declination respectively of a star, the true right ascension and declination being given thus:

$a = a_1$ plus the correction for transformation.

$\delta = \delta_1$ plus the correction for transformation.

Then collecting the corrections given above:

$$\begin{array}{l} a_1 = (1 + p + M_x) X \sec \delta_0 + (N_x + r \sec \delta_0) Y + (a_0 + k \sec \delta_0) \\ \delta_1 = (1 + p + N_y) Y \quad \quad + (M_y - r \cos \delta_0) X \sec \delta_0 + (\delta_0 + c) \end{array}$$

Hence, to get the projected right ascension and declination of any star, the constants of the plate having been determined, we need only compute the six coefficients in the parentheses and perform the simple operations indicated. These coefficients, it is needless to remark, are constant for an entire plate.

As an example of the above methods I have set down the details of the computations for the constants of Plate VIII.

Star	Right Ascensions.				
	4	5	15	40	44
$x : (\text{Table V}),$	-30.6580	-21.5809	0	+25.9804	+34.4505
$X \sec \delta_0 = 52.87 \sec \delta_0 \cdot x,$	-1727.31	-1215.89	0	+1463.77	+1940.98
$M_x \cdot X \sec \delta_0 : (\text{Tab. VII}),$	-0.85	-.60	0	+0.72	+0.95
$N_x \cdot Y : (\text{See below}),$	-0.14	+.12	0	-0.14	+0.11
$\Delta a - X \sec \delta_0 : (\text{Tab. VI}),$	+5.75	-3.33	0	-4.84	+4.79
$-\Delta a : (\text{Page 237}),$	+1723.40	+1219.75	0	-1459.48	-1948.19
$n_x \sec \delta_0 :$	+0.85	+0.05	0	+0.03	-1.36
$n_x :$	+0.80	+0.05	0	+0.03	-1.28

Star.	Declinations.				
	4	5	15	40	44
$y : (\text{Table V}),$	-35.0520	+28.9675	0	-34.9240	+26.3259
$Y = 52.87 y :$	-1853.20	+1531.51	0	-1846.43	+1391.85
$N_y \cdot Y : (\text{Table VII}),$	-0.75	+0.62	0	-0.75	+0.56
$N_x \cdot X \sec \delta_0 : (\text{See above}),$	-0.40	-0.28	0	+0.34	+0.46
$\Delta \delta - Y : (\text{Table VI}),$	-2.23	-1.22	0	-1.59	-3.05
$-\Delta \delta : (\text{Page 237}),$	+1856.81	-1531.17	0	+1849.40	-1390.15
$n_y :$	+0.23	-0.54	0	+0.97	-0.33

$$[X \cdot n_x] = -3650 \quad [Y \cdot n_x] = -3240 \quad [n_x] = -0''.40$$

$$[X \cdot n_y] = +980 \quad [Y \cdot n_y] = -3510 \quad [n_y] = +0''.33$$

$$p = +0.000352 \quad \pm 0.000030$$

$$r = +0.000211 \quad \pm 0.000030$$

$$k = +0''.08 \quad \pm 0''.061$$

$$c = +0''.01 \quad \pm 0''.061$$

Having found the constants, we may now proceed to get the right ascension and declination of each star on the plate. For this purpose we compute the coefficients:

$$\begin{array}{ll} \text{For Right Ascensions.} & \text{For Declinations.} \\ 52.87 (1 + p + M_x) \sec \delta_0 = 56.3887 & 52.87 (1 + p + N_y) = 52.9100 \\ N_x + r \sec \delta_0 = +0.000299 & M_y - r \cos \delta_0 = +0.000035 \\ a_0 + k \sec \delta_0 = 128^\circ 07' 54''.84 & \delta_0 + c = 20^\circ 13' 01''.39 \end{array}$$

A slight change has been made in the two coefficients in the first line, our formulas requiring $(1 + p + M_x)$ and $(1 + p + N_y)$. This change, however, merely amounts to combining two multiplications into one; thus, instead of first computing $X \sec \delta_0$ and then multiplying this by $(1 + p + M_x)$, we may apply the factor $52.87(1 + p + M_x) \sec \delta_0$ to x directly. The formulas require that we

know $X \sec \delta_0$ in computing the correction to the declination, but for this purpose we may use $52.87(1+p+M_x) \sec \delta_0 \cdot x$, the quantity which we have just computed and which is practically equal to $X \sec \delta_0$ for this purpose. Similarly in the declination we may compute at once $52.87(1+p+N_y)$.

Employing these coefficients for Star 7 we have:

Right Ascension:		Declination:	
$x : (\text{Tab. V}),$	— 14.6960	$y : (\text{Tab. V}),$	— 19.0252
$52.87(1+p+M_x) \sec \delta_0 \cdot x : —$	828."69	$52.87(1+y+N_y) \cdot y : —$	1006".62
	— 13'48.69		— 16'46".62
$(N_x+r \sec \delta_0) \cdot Y :$	— 0.30	$(M_y-r \sec \delta_0) \cdot X \sec \delta_0 : —$	0.03
$a_0+k \sec \delta_0 :$	128°07 54 .84	$\delta_0+c :$	+20°13 01 .39
$a_1 =$	127°54'05".85	$\delta_1 =$	+19°56'14".74

These give the projected position; the true right ascension and declination are found by adding the transformation corrections from Table VI, giving:

$$\alpha = 127^\circ 54' 7''.34, \quad \delta = +19^\circ 56' 14''.22,$$

which are corrected for refraction, precession, nutation and aberration, and are referred to the mean equinox of 1875.0.

V.

Results.

Least-square solutions entirely similar to that given in detail in the last section lead to the following values of p , r , etc., for the various plates :

CONSTANTS OF THE PLATES.

Plate.	p	r	Probable Error of p or r .	k	c	Probable Error of k or c .
I.	+0.000100	+0.001054	± 0.000037	—0.06	0.00	± 0.075
II.	+ 125	+ 844	± 45	—0.18	—0.15	± 0.092
III.	+ 125	+ 409	± 32	—0.22	—0.06	± 0.066
IV.	+ 209	+ 1435	± 41	0.00	—0.03	± 0.082
V.	+ 261	+ 172	± 22	—0.01	+0.01	± 0.044
VII.	+ 342	— 35	± 24	+0.02	—0.04	± 0.049
VIII.	+ 352	+ 211	± 30	+0.08	+0.01	± 0.061
IX.	+ 365	— 326	± 24	—0.06	—0.01	± 0.048

The average probable error of p or r is

$$\pm 0.000032$$

which corresponds to an uncertainty of about 0''.06 in the coördinates of the outlying stars. The great diversity in the values of r is due in small part to corrections for precession, nutation and aberration, but chiefly to the accidental position in which the plate was inserted in the measuring machine.

The following are the residuals for the five comparison stars, used in computing the values of p , r , k and c given above.

Residuals from the Right Ascension Equations :

Plate.	Star 4.	Star 5.	Star 15.	Star 40.	Star 44.
I	0.00	+0.03	—0.01	+0.07	—0.09
II	— .24	+ .31	— .18	+ .42	— .31
III	— .14	+ .20	— .23	+ .28	— .11
IV	— .19	+ .13	.00	+ .39	— .33
V	— .01	+ .13	— .06	+ .25	— .31
VII	— .07	+ .09	+ .02	+ .14	— .18
VIII	— .08	+ .05	+ .08	+ .21	— .26
IX	+ .10	— .09	— .06	+ .12	— .08
Means,	— .08	+ .11	— .06	+ .24	— .21

Residuals from the Declination Equations:

Plate.	Star 4.	Star 5.	Star 15.	Star 40.	Star 44.
I	—0.13	+0.20	+0.01	+0.10	—0.18
II	— .03	+ .20	— .15	+ .11	— .13
III	— .07	+ .13	— .06	+ .16	— .16
IV	— .00	+ .23	— .04	+ .07	— .26
V	— .09	+ .34	.00	— .01	— .24
VII	— .10	+ .22	— .04	+ .08	— .16
VIII	— .08	+ .25	+ .01	+ .04	— .22
IX	— .02	+ .23	— .01	+ .03	— .23
Means,	— .06	+ .22	— .03	+ .07	— .20

Employing the constants in the manner described at length in the last section, we obtain the quantities α_1 and δ_1 which have been tabulated in the following pages. It will be remembered that α_1 and δ_1 are the projected right ascension and declination of a star respectively; the transformation correction being the same for all the plates, may just as well be applied to the means; and it is evident that this procedure does not affect in any way the comparison of the right ascensions or declinations of a star as derived from different plates. The columns headed "At Epoch of Plate" give the coördinates uncorrected for proper motion. The calculation of the latter is very simple in this case as the plates were taken at practically only two dates, 1870.3 and 1877.3; hence the annual proper motion is obtained by subtracting the mean of the places on plates of the earlier date from the mean for the later date, and dividing the difference by 7. The columns marked "P. M." give the correction for proper motion necessary to reduce the place of the star to the epoch 1875.0.

Probable errors are given for the right ascension and the declination of each star, and also for the proper motions; they were calculated thus:

Let m = the number of plates of date 1870.3 on which the star was measured.

n = the number of date 1877.3.

$[vv]$ = the sum of the squares of the residuals obtained by subtracting the mean from the separate observations reduced to the epoch 1875.0.

Then the probable error of a quantity having the weight unity is:

$$\pm 0.6745 \sqrt{\frac{[vv]}{m+n-2}}$$

The weight of a right ascension or a declination at the epoch 1875.0 is

$$\frac{49mn}{22.1 m + 5.3 n}$$

For a proper motion the weight is

$$\frac{49mn}{m+n}$$

Most of the stars appear on all eight plates ; for such we have simply,

$$\begin{aligned} \text{Probable error of } a_1 \text{ or of } \delta_1 &= \pm 0.103 \sqrt{[vv]} \\ \text{" " of proper motion} &= \pm 0.028 \sqrt{[vv]} \end{aligned}$$

Two of the stars were observed only on plates taken in 1877, and consequently it was not possible to reduce them to the epoch 1875.0 by using proper motions determined from the plates themselves, as has been done for all the other stars. The proper motions used for these two stars are those given by Professor Schur on page 298 of his memoir, and are as follows :

Star.	P. M. in Right Ascension.	P. M. in Declination.
41	—0".042	+0".012
42	—0 .066	+0 .036

These were used for an interval of only 2.3 years.

STAR I.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	127°35'31''.25	—0''.43	30''.82	20°26'52''.60	+0''.15	52''.75
II	30 .80	— .43	.37	.47	+ .15	.62
III	30 .94	— .43	.51	.52	+ .15	.67
IV	31 .16	— .43	.73	.64	+ .15	.79
V	30 .46	+ .21	.67	.84	— .07	.77
VII	30 .47	+ .21	.68	.79	— .07	.72
VIII	30 .45	+ .21	.66	.60	— .07	.53
IX	30 .22	+ .21	.43	.90	— .07	.83
Mean,			127°35'30''.61	Mean,		
Probable Error,			±.041	20°26'52''.71		
Probable Error,				±.026		
Proper Motion,			—0.091 ±0.011	Proper Motion, +0.032 ±0.007		

STAR 2.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	127°36'26''.17	—0''.31	25''.86	20°10'02''.04	+0''.15	2''.19
II	26 .21	— .31	.90	.10	+ .15	.25
III	26 .03	— .31	.72	.06	+ .14	.20
IV	26 .15	— .31	.84	.14	+ .14	.28
V	25 .74	+ .15	.89	.26	— .07	.19
VII	25 .65	+ .15	.80	.32	— .07	.25
VIII	25 .60	+ .15	.75	.24	— .07	.17
IX	25 .71	+ .15	.86	.40	— .07	.33
Mean,			127°36'25''.83	Mean,		
Probable Error,			±.018	20°10'02''.23		
Probable Error,				±.015		
Proper Motion,			—0.066 ±0.005	Proper Motion, +0.031 ±0.004		

STAR 3.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°37'28''.20	—0''.38	27''.82	19°57'10''.51	+0''.01	10''.52	
III	27 .95	— .38	.57	.32	+ .01	.33	
IV	27 .98	— .38	.60	.42	+ .01	.43	
V	27 .53	+ .19	.72	.50	.00	.50	
VII	27 .39	+ .19	.58	.39	.00	.39	
VIII	27 .36	+ .19	.55	.34	.00	.34	
IX	27 .62	+ .19	.81	.49	— .01	.48	
Mean,			127°37'27''.66	Mean,			19°57'10''.43
Probable Error,			±.032	Probable Error,			±.021
Proper Motion,			—0.081 ±0.009	Proper Motion,			+0.002 ±0.006

STAR 4.

Plate.	Right Ascension:			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	127°39'06''.16	—0''.29	5''.87	19°42'06''.67	+0''.01	6''.68
II	5 .92	— .29	.63	.72	+ .01	.73
III	6 .04	— .29	.75	.68	+ .01	.69
IV	5 .97	— .29	.68	.74	+ .01	.75
V	5 .60	+ .14	.74	.66	.00	.66
VII	5 .54	+ .14	.68	.70	.00	.70
VIII	5 .52	+ .14	.66	.73	.00	.73
IX	5 .71	+ .14	.85	.77	— .01	.76
Mean,			127°39'05''.73	Mean, 19°42'06''.71		
Probable Error,			±.024	Probable Error, ±.010		
Proper Motion,			—0.061 ±0.007	Proper Motion, +0.002 ±0.003		

STAR 5.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°47'38''.48	—0''.14	8''.34	20°38'33''.84	+0''.17	34''.01	
II	.68	— .14	.54	33 .71	+ .17	33 .88	
III	.57	— .14	.43	33 .64	+ .17	33 .81	
IV	.48	— .14	.34	33 .74	+ .17	33 .91	
V	.35	+ .07	.42	33 .97	— .08	33 .89	
VII	.41	+ .07	.48	33 .99	— .09	33 .90	
VIII	.38	+ .07	.45	34 .01	— .09	33 .92	
IX	.22	+ .07	.29	34 .01	— .09	33 .92	
Mean,			127°47'38''.41	Mean,			20°38'33''.91
Probable Error,			±.022	Probable Error,			±.015
Proper Motion,			—0.030 ±0.006	Proper Motion,			+0.037 ±0.004

STAR 6.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°51'33''.81	—0''.32	33''.49	20°25'33''.46	+0''.21	33''.67	
II	.66	— .32	.34	.16	+ .21	.37	
III	.77	— .32	.45	.30	+ .21	.51	
IV	.97	— .32	.65	.52	+ .21	.73	
V	.47	+ .16	.63	.66	— .10	.56	
VII	.34	+ .16	.50	.61	— .10	.51	
VIII	.26	+ .16	.42	.64	— .10	.54	
IX	.20	+ .16	.36	.76	— .10	.66	
Mean,			127°51'33''.48	Mean,			20°25'33''.57
Probable Error,			±.031	Probable Error,			±.031
Proper Motion,			—0.069 ±0.009	Proper Motion,			+0.044 ±0.009

STAR 7.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	127°54'06''.35	—0''.33	6''.02	19°56'14''.87	—0''.03	14''.84
II	6 .36	— .33	6 .03	.70	— .03	.67
III	6 .16	— .33	5 .83	.63	— .03	.60
IV	6 .46	— .33	6 .13	.88	— .03	.85
V	5 .91	+ .16	6 .07	.76	+ .01	.77
VII	5 .81	+ .16	5 .97	.74	+ .01	.75
VIII	5 .85	+ .16	6 .01	.74	+ .01	.75
IX	5 .77	+ .17	5 .94	.66	+ .01	.67
Mean,			127°54'06''.00	Mean, 19°56'14''.74		
Probable Error,			±.025	Probable Error, ±.024		
Proper Motion,			—0.071 ±0.007	Proper Motion, —0.006 ±0.007		

STAR 7A.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
II	127°57'58''.00	—0''.47	57''.53	20°09'42''.62	—0''.09	42''.53
IV	58 .26	— .47	.79	.55	— .09	.46
VII	57 .61	+ .23	.84	.57	+ .04	.61
VIII	57 .25	+ .23	.48	.34	+ .04	.38
Mean,			127°57'57''.66	Mean, 20°09'42''.50		
Proper Motion,			—0.100	Proper Motion, —0.019		

STAR 8.

	Right Ascension :			Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°57'59''.38	—0''.34	9''.04	20°26'12''.54	+0''.12	12''.66	
II	59 .38	— .34	.04	.51	+ .12	.63	
III	59 .64	— .34	.30	.63	+ .12	.75	
IV	59 .42	— .34	.08	.69	+ .12	.81	
V	59 .08	+ .17	.25	.79	— .06	.73	
VII	58 .90	+ .17	.07	.69	— .06	.63	
VIII	58 .86	+ .17	.03	.70	— .06	.64	
IX	58 .95	+ .17	.12	.88	— .06	.82	
Mean,			127°57'59''.12	Mean,			20°26'12''.71
Probable Error,			±.028	Probable Error,			±.022
Proper Motion,			—0.072 ±0.008	Proper Motion,			+0.025 ±0.006

STAR 10.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°58'48''.65	—0''.19	48''.46	20°06'50''.40	+0''.14	50''.54	
II	.76	— .19	.57	.33	+ .14	.47	
III	.83	— .19	.64	.27	+ .14	.41	
IV	.77	— .19	.58	.34	+ .14	.48	
V	.47	+ .09	.56	.52	— .07	.45	
VII	.34	+ .09	.43	.58	— .07	.51	
VIII	.42	+ .09	.51	.48	— .07	.41	
IX	.67	+ .09	.76	.56	— .07	.49	
Mean,			127°58'48''.56	Mean,			20°06'50''.47
Probable Error,			±.028	Probable Error,			±.012
Proper Motion,			—0.040 ±0.008	Proper Motion,			+0.029 ±0.003

STAR II.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	127°59'36''.89	—0''.35	36''.54	20°01'46''.90	+0''.10	47''.00	
II	36 .86	— .35	.51	46 .86	+ .10	46 .96	
III	36 .96	— .35	.61	46 .80	+ .10	46 .90	
IV	37 .08	— .35	.73	46 .76	+ .10	46 .86	
V	36 .63	+ .17	.80	46 .95	— .05	46 .90	
VII	36 .36	+ .17	.53	46 .96	— .05	46 .91	
VIII	36 .40	+ .17	.57	46 .83	— .05	46 .78	
IX	36 .29	+ .17	.46	47 .16	— .05	47 .11	
Mean,			127°59'36''.59	Mean,			20°01'46''.93
Probable Error,			±.032	Probable Error,			±.027
Proper Motion,			—0.075 ±0.009	Proper Motion,			+0.021 ±0.008

STAR 14.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°07'45''.06	—0''.20	44''.86	20°31'29''.10	+0''.13	29''.23	
II	44 .81	— .20	.61	29 .00	+ .13	.13	
III	44 .98	— .20	.78	28 .88	+ .13	.01	
IV	45 .15	— .20	.95	29 .01	+ .13	.14	
V	44 .82	+ .10	.92	29 .07	— .06	.01	
VII	44 .60	+ .10	.70	29 .19	— .06	.13	
VIII	44 .75	+ .10	.85	29 .23	— .06	.17	
IX	44 .64	+ .10	.74	29 .26	— .06	.20	
Mean,			128°07'44''.80	Mean,			20°31'29''.13
Probable Error,			±.031	Probable Error,			±.022
Proper Motion,			—0.042 ±0.009	Proper Motion,			+0.027 ±0.006

STAR 15.

Right Ascension :				Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°07'55''.19	—0''.25	54''.94	20°13'01''.26	+0''.12	1''.38	
II	55 .07	— .25	54 .82	.11	+ .12	.23	
III	55 .02	— .25	54 .77	.20	+ .12	.32	
IV	55 .26	— .25	55 .01	.23	+ .12	.35	
V	54 .74	+ .12	54 .86	.39	— .06	.33	
VII	54 .77	+ .12	54 .89	.34	— .06	.28	
VIII	54 .84	+ .12	54 .96	.39	— .06	.33	
IX	54 .69	+ .13	54 .82	.37	— .06	.31	
Mean,			128°07'54''.88	Mean,			20°13'01''.32
Probable Error,			±.022	Probable Error,			±.012
Proper Motion,			—0.054 ±0.006	Proper Motion,			+0.025 ±0.003

STAR 16.

	Right Ascension :			Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°08'44''.80	—0''.34	44''.46	19°42'50''.00	+0''.03	50''.03	
II	.79	— .34	.45	50 .08	+ .03	50 .11	
III	.73	— .34	.39	49 .91	+ .03	49 .94	
IV	.98	— .34	.64	49 .85	+ .03	49 .88	
V	.38	+ .17	.55	49 .97	— .01	49 .96	
VII	.41	+ .17	.58	50 .00	— .01	49 .99	
VIII	.32	+ .17	.49	49 .95	— .01	49 .94	
IX	.15	+ .17	.32	50 .08	— .01	50 .07	
Mean,			128°08'44''.48	Mean,			19°42'49''.99
Probable Error,			±.028	Probable Error,			±.021
Proper Motion,			—0.073 ±0.008	Proper Motion,			+0.006 ±0.006

STAR 17.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 10' 05''.46	— 0''.27	5''.19	19° 48' 46''.81	+ 0''.09	46''.90	
II	5 .17	— .27	4 .90	.83	+ .09	.92	
III	5 .36	— .27	5 .09	.81	+ .08	.89	
IV	5 .57	— .27	5 .30	.75	+ .08	.83	
V	5 .03	+ .13	5 .16	.88	— .04	.84	
VII	5 .06	+ .13	5 .19	.90	— .04	.86	
VIII	5 .01	+ .13	5 .14	.93	— .04	.89	
IX	4 .83	+ .14	4 .97	.99	— .04	.95	
Mean, Probable Error,			128° 10' 05''.12 ±.035	Mean, Probable Error,			19° 48' 46''.89 ±.011
Proper Motion,			—0.058 ±0.010	Proper Motion,			+0.018 ±0.003

STAR 18.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 11' 32''.44	— 0''.22	32''.22	19° 59' 31''.64	+ 0''.10	31''.74	
II	32 .26	— .22	32 .04	.53	+ .10	.63	
III	32 .20	— .21	31 .99	.44	+ .10	.54	
IV	32 .28	— .21	32 .07	.48	+ .10	.58	
V	32 .05	+ .11	32 .16	.74	— .05	.69	
VII	31 .95	+ .11	32 .06	.60	— .05	.55	
VIII	32 .04	+ .11	32 .15	.64	— .05	.59	
IX	31 .86	+ .11	31 .97	.69	— .05	.64	
Mean,			128° 11' 32''.08	Mean,			19° 59' 31''.62
Probable Error,			±.024	Probable Error,			±.019
Proper Motion,			—0.046 ±0.007	Proper Motion,			+0.021 ±0.005

STAR 19.

Right Ascension:				Declination:			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
II	128°11'36''.06	—0''.25	35''.81	20°35'50''.68	+0''.09	50''.77	
IV	36 .37	— .25	36 .12	.64	+ .09	.73	
V	35 .95	+ .12	36 .07	.69	— .04	.65	
VII	35 .86	+ .12	35 .98	.87	— .04	.83	
VIII	35 .72	+ .12	35 .84	.82	— .04	.78	
Mean,			128°11'35''.96	Mean, 20°35'50''.75			
Proper Motion,			—0.053	Proper Motion, +0.019			

STAR 20.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°11'51''.88	—0''.20	51''.68	19°58'50''.40	+0''.08	50''.48
II	.77	— .20	.57	.42	+ .08	.50
III	.75	— .20	.55	.39	+ .08	.47
IV	.77	— .20	.57	.40	+ .08	.48
V	.49	+ .10	.59	.50	— .04	.46
VII	.42	+ .10	.52	.37	— .04	.33
VIII	.51	+ .10	.61	.50	— .04	.46
IX	.58	+ .10	.68	.68	— .04	.64
Mean,			128°11'51''.60	Mean, 19°58'50''.48		
Probable Error,			±.016	Probable Error, ±.023		
Proper Motion,			—0.042 ±0.004	Proper Motion, +0.016 ±0.006		

STAR 22.

Right Ascension :				Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 12' 43''.03	— 0''.32	42''.71	20° 26' 50''.07	+ 0''.13	50''.20	
II	42 .70	— .32	.38	.12	+ .13	.25	
III	42 .98	— .32	.66	.04	+ .13	.17	
IV	43 .08	— .32	.76	.04	+ .13	.17	
V	42 .36	+ .16	.52	.20	— .06	.14	
VII	42 .33	+ .16	.49	.20	— .06	.14	
VIII	42 .57	+ .16	.73	.35	— .06	.29	
IX	42 .63	+ .16	.79	.28	— .06	.22	
Mean,			128° 12' 42''.63	Mean, 20° 26' 50''.20			
Probable Error,			±.040	Probable Error, ±.015			
Proper Motion,			—0.068 ±0.011	Proper Motion, +0.027 ±0.004			

STAR 23.

	Right Ascension :			Declination :		
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Place.	P. M.	At Epoch 1875.0.
I	128° 14' 60''.24	— 0''.23	60''.01	20° 34' 39''.08	+ 0''.19	39''.27
II	59 .92	— .23	59 .69	38 .93	+ .19	.12
III	59 .81	— .23	59 .58	38 .91	+ .19	.10
IV	60 .20	— .23	59 .97	38 .94	+ .19	.13
V	59 .85	+ .11	59 .96	39 .21	— .09	.12
VII	59 .54	+ .11	59 .65	39 .18	— .09	.09
VIII	59 .71	+ .11	59 .82	39 .18	— .09	.09
IX	59 .69	+ .11	59 .80	39 .41	— .09	.32
Mean,			128° 14' 59''.81	Mean, 20° 34' 39''.16		
Probable Error,			±.044	Probable Error, ±.025		
Proper Motion,			—0.049 ±0.012	Proper Motion, +0.040 ±0.007		

STAR 23A.

Right Ascension :				Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 15' 57".80	— 0".32	57".48	20° 26' 03".10	+ 0".09	3".19	
II	.74	— .32	.42	2 .80	+ .09	2 .89	
III	.70	— .32	.38	3 .00	+ .09	3 .09	
IV	.96	— .32	.64	2 .96	+ .09	3 .05	
V	.42	+ .16	.58	3 .10	— .05	3 .05	
VII	.28	+ .16	.44	3 .09	— .05	3 .04	
VIII	.27	+ .16	.43	3 .12	— .05	3 .07	
IX	.29	+ .16	.45	3 .12	— .05	3 .07	
Mean,			128° 15' 57".48	Mean,			20° 26' 03".06
Probable Error,			±.024	Probable Error,			±.022
Proper Motion,			— 0.069 ± 0.007	Proper Motion,			+ 0.020 ± 0.006

STAR 24.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°16'58''.57	—0''.21	58''.36	19°58'19''.27	+0''.07	19''.34	
II	58 .24	— .21	58 .03	.16	+ .07	.23	
III	58 .57	— .21	58 .36	.22	+ .07	.29	
IV	58 .67	— .21	58 .46	.11	+ .07	.18	
V	58 .19	+ .10	58 .29	.29	— .03	.26	
VII	57 .89	+ .10	57 .99	.24	— .03	.21	
VIII	58 .32	+ .10	58 .42	.27	— .03	.24	
IX	58 .38	+ .10	58 .48	.38	— .04	.34	
Mean,			128°16'58''.30	Mean,			19°58'19''.26
Probable Error,			±.051	Probable Error,			±.016
Proper Motion,			—0.045 ±0.014	Proper Motion,			+0.015 ±0.004

STAR 25.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 17' 32'' .03	— 0'' .25	31'' .78	20° 07' 36'' .79	+ 0'' .07	36'' .86	
II	31 .70	— .25	.45	.67	+ .07	.74	
III	31 .57	— .25	.32	.60	+ .07	.67	
IV	31 .95	— .25	.70	.69	+ .07	.76	
V	31 .39	+ .12	.51	.73	— .03	.70	
VII	31 .38	+ .12	.50	.82	— .03	.79	
VIII	31 .38	+ .12	.50	.75	— .03	.72	
IX	31 .59	+ .13	.72	.86	— .04	.82	
Mean,			128° 17' 31'' .56	Mean,			20° 07' 36'' .76
Probable Error,			± .043	Probable Error,			± .016
Proper Motion,			— 0.054 ± 0.012	Proper Motion,			+ 0.015 ± 0.004

STAR 26.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128° 17' 37''.23	— 0''.30	36''.93	19° 47' 20''.91	+ 0''.03	20''.94
II	36 .92	— .30	.62	21 .01	+ .03	21 .04
III	36 .98	— .29	.69	20 .83	+ .03	20 .86
IV	37 .12	— .29	.83	20 .76	+ .03	20 .79
V	36 .49	+ .14	.63	20 .82	— .02	20 .80
VII	36 .64	+ .15	.79	20 .90	— .02	20 .88
VIII	36 .79	+ .15	.94	20 .89	— .02	20 .87
IX	36 .56	+ .15	.71	21 .09	— .02	21 .07
Mean,			128° 17' 36''.77	Mean, 19° 47' 20''.91		
Probable Error,			±.034	Probable Error, ±.028		
Proper Motion,			—0.063 ±0.009	Proper Motion, +0.007 ±0.008		

STAR 27.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128° 17' 52''.80	— 0''.17	52''.63	20° 06' 36''.49	+ 0''.15	36''.64
II	.35	— .17	.18	.67	+ .15	.82
III	.56	— .16	.40	.50	+ .15	.65
IV	.71	— .16	.55	.43	+ .15	.58
V	.42	+ .08	.50	.70	— .07	.63
VII	.34	+ .08	.42	.73	— .07	.66
VIII	.30	+ .08	.38	.71	— .07	.64
IX	.39	+ .08	.47	.86	— .08	.78
Mean,			128° 17' 52''.44	Mean, 20° 06' 36''.68		
Probable Error,			±.037	Probable Error, ±.022		
Proper Motion,			—0.035 ±0.010	Proper Motion, +0.032 ±0.006		

STAR 28.

Right Ascension :				Declination :		
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°18'06''.53	—0''.19	6''.34	20°16'31''.01	+0''.07	31''.08
II	.26	— .19	.07	30 .96	+ .07	31 .03
III	.44	— .19	.25	30 .78	+ .07	30 .85
IV	.58	— .19	.39	30 .93	+ .07	31 .00
V	.21	+ .09	.30	30 .98	— .03	30 .95
VII	.03	+ .09	.12	30 .97	— .03	30 .94
VIII	.22	+ .09	.31	31 .08	— .03	31 .05
IX	.22	+ .09	.31	31 .06	— .04	31 .02
Mean,			128°18'06''.26	Mean, 20°16'30''.99		
Probable Error,			±.030	Probable Error, ±.020		
Proper Motion,			—0.040 ±0.008	Proper Motion, +0.015 ±0.006		

STAR 29.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°18'40''.95	—0''.29	40''.66	20''.52	+0''.17	20''.69
II	40 .52	— .29	.23	.31	+ .17	.48
III	40 .87	— .29	.58	.40	+ .17	.57
IV	41 .08	— .29	.79	.37	+ .17	.54
V	40 .50	+ .14	.64	.48	— .08	.40
VII	40 .30	+ .14	.44	.71	— .08	.63
VIII	40 .50	+ .14	.64	.62	— .08	.54
IX	40 .38	+ .14	.52	.81	— .08	.73
Mean,			128°18'40''.56	Mean,		
Probable Error,			±.046	20°37'20''.57		
Probable Error,				±.030		
Proper Motion,			—0.062 0.012	Proper Motion, +0.036 ±0.008		

STAR 31.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°19'12''.34	—0''.27	12''.07	19°59'06''.84	+0''.10	6''.94
II	12 .31	— .27	12 .04	6 .88	+ .10	6 .98
III	11 .93	— .27	11 .66	6 .93	+ .10	7 .03
IV	12 .18	— .27	12 .91	6 .64	+ .10	6 .74
V	11 .64	+ .13	11 .77	6 .94	— .05	6 .89
VII	11 .73	+ .13	11 .86	7 .02	— .05	6 .97
VIII	11 .95	+ .13	12 .08	6 .88	— .05	6 .83
IX	11 .83	+ .13	11 .96	7 .07	— .05	7 .02
Mean,			128°19'11''.92	Mean,		
Probable Error,			±.041	19°59'06''.92		
Probable Error,				±.027		
Proper Motion,			—0.057 ±0.011	Proper Motion, +0.022 ±0.008		

STAR 32.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°20'56''.93	—0''.28	56''.65	19°38'06''.20	+0''.03	6''.23
II	57 .02	— .28	.74	.35	+ .03	.38
III	56 .98	— .28	.70	.17	+ .03	.20
IV	57 .20	— .28	.92	.05	+ .03	.08
V	56 .51	+ .14	.65	.28	— .02	.26
VII	56 .61	+ .14	.75	.27	— .02	.25
VIII	56 .71	+ .14	.85	.11	— .02	.09
IX	56 .64	+ .14	.78	.32	— .02	.30
Mean,			128°20'56''.76	Mean, 19°38'06''.22		
Probable Error,			±.026	Probable Error, ±.028		
Proper Motion,			—0.059 ±0.007	Proper Motion, +0.007 ±0.008		

STAR 33.

Right Ascension :				Declination :			
Plate.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°22'30''.50	—0''.34	30''.16	19°40'09''.34	+0''.04	9''.38	
II	30 .37	— .34	30 .03	.57	+ .04	.61	
III	30 .14	— .34	29 .80	.34	+ .04	.38	
IV	30 .53	— .34	30 .19	.32	+ .04	.36	
VII	29 .92	+ .17	30 .09	.50	— .02	.48	
VIII	30 .02	+ .17	30 .19	.29	— .02	.27	
IX	29 .71	+ .17	29 .88	.57	— .02	.55	
Mean,			128°22'30''.05	Mean,			19°40'09''.43
Probable Error,			±.048	Probable Error,			±.037
Proper Motion,			—0.072 ±0.013	Proper Motion,			+0.009 ±0.010

STAR 34.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°23'08''.33	—0''.27	8''.06	20°09'37''.37	+0''.10	37''.47	
II	8 .29	— .27	8 .02	.32	+ .10	.42	
III	8 .33	— .27	8 .06	.30	+ .10	.40	
IV	8 .49	— .27	8 .22	.36	+ .10	.46	
V	7 .80	+ .13	7 .93	.44	— .05	.39	
VII	8 .06	+ .13	8 .19	.58	— .05	.53	
VIII	8 .08	+ .13	8 .21	.48	— .05	.43	
IX	7 .88	+ .14	8 .02	.47	— .05	.42	
Mean,			128°23'08''.09	Mean,			20°09'37''.44
Probable Error,			±.029	Probable Error,			±.012
Proper Motion,			—0.058 ±0.008	Proper Motion,			+0.022 ±0.003

STAR 35.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°25'03''.01	—0''.27	2''.74	20°42'28''.84	+0''.14	28''.98	
II	2 .92	— .27	.65	28 .80	+ .14	28 .94	
III	3 .04	— .27	.77	28 .86	+ .14	29 .00	
IV	3 .01	— .27	.74	28 .82	+ .14	28 .96	
V	2 .78	+ .13	.91	28 .86	— .07	28 .79	
VII	2 .58	+ .13	.71	29 .12	— .07	29 .05	
VIII	2 .59	+ .13	.72	29 .00	— .07	28 .93	
IX	2 .41	+ .14	.55	29 .17	— .07	29 .10	
Mean,			128°25'02''.72	Mean,			20°42'28''.97
Probable Error,			±.028	Probable Error,			±.025
Proper Motion,			—0.058 ±0.008	Proper Motion,			+0.030 ±0.007

STAR 36.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°26'25''.18	—0''.45	24''.73	20°22'35''.73	+0''.16	35''.89
II	25 .14	— .45	.69	.67	+ .16	.83
III	25 .17	— .44	.73	.59	+ .15	.74
IV	25 .40	— .44	.96	.63	+ .15	.78
VII	24 .59	+ .22	.81	.96	— .08	.88
VIII	24 .54	+ .22	.76	.78	— .08	.70
IX	24 .54	+ .22	.76	.92	— .08	.84
Mean,		128°26'24''.78		Mean,		20°22'35''.81
Probable Error,		±.028		Probable Error,		±.022
Proper Motion,		—0.095 +0.007		Proper Motion,		+0.033 ±0.006

STAR 37.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°26'31''.81	—0''.28	31''.53	20°01'19''.38	+0''.05	19''.43
II	.61	— .28	.33	.56	+ .05	.61
III	.50	— .28	.22	.31	+ .05	.36
IV	.89	— .28	.61	.28	+ .05	.33
V	.28	+ .14	.42	.42	— .02	.40
VII	.29	+ .14	.43	.34	— .03	.31
VIII	.32	+ .14	.46	.49	— .03	.46
IX	.27	+ .14	.41	.58	— .03	.55
Mean,		128°26'31''.43		Mean,		20°01'19''.43
Probable Error,		±.032		Probable Error,		±.029
Proper Motion,		—0.059 ±0.006		Proper Motion,		+0.011 ±0.008

STAR 38.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
I	128°29'56''.44	—0''.21	56''.23	20°16'19''.07	+0''.08	19''.15
II	.48	— .21	.27	19 .16	+ .08	.24
IV	.66	— .21	.45	18 .93	+ .07	.00
VII	.17	+ .10	.27	19 .08	— .04	.04
VIII	.15	+ .10	.25	19 .17	— .04	.13
IX	.32	+ .10	.42	19 .24	— .04	.20
Mean,			128°29'56''.32	Mean, 20°16'19''.13		
Probable Error,			±.031	Probable Error, ±.030		
Proper Motion,			—0.045 ±0.008	Proper Motion, +0.016 ±0.008		

STAR 39.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128° 30' 39''.00	— 0''.35	38''.65	20° 21' 52''.34	+ 0''.07	52''.41	
II	38 .72	— .35	.37	.34	+ .07	.41	
III	38 .82	— .35	.47	.32	+ .07	.39	
IV	39 .01	— .35	.66	.24	+ .07	.31	
V	38 .44	+ .17	.61	.30	— .03	.27	
VII	38 .33	+ .17	.50	.42	— .03	.39	
VIII	38 .34	+ .17	.51	.38	— .03	.35	
IX	38 .33	+ .17	.50	.56	— .04	.52	
Mean,			128° 30' 38''.53	Mean,			20° 21' 52''.38
Probable Error,			±.027	Probable Error,			±.021
Proper Motion,			—0.075 ±0.008	Proper Motion,			+0.015 ±0.006

STAR 40.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875 0.	At Epoch of Plate.	P. M.	At Epoch 1875 0.	
I	128°32'19''.74	—0''.42	19''.32	19°42'13''.48	+0''.05	13''.53	
II	.92	— .42	.50	.60	+ .05	.65	
III	.78	— .42	.36	.65	+ .05	.70	
IV	.92	— .42	.50	.54	+ .05	.59	
V	.14	+ .21	.35	.67	— .02	.65	
VII	.22	+ .21	.43	.65	— .02	.63	
VIII	.29	+ .21	.50	.61	— .02	.59	
IX	.20	+ .21	.41	.61	— .02	.59	
Mean,			128°32'19''.42	Mean,			19°42'13''.62
Probable Error,			±.020	Probable Error,			±.014
Proper Motion,			—0.090 ±0.006	Proper Motion,			+0.010 ±0.004

STAR 41.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875 0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
VII	128°06'08''.44	+0''.10	8''.54	20°06'21''.68	—0''.03	21''.65
VIII	.30	+ .10	.40	.78	— .03	.75
Mean,			128°06'08''.47	Mean, 20°06'21''.70		

STAR 42.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
VII	128°38'00''.16	+0''.15	00''.31	20°06'16''.46	—0''.08	16''.38
VIII	59 .83	+ .15	59 .98	.29	— .08	.21
Mean,			128°38'00''.14	Mean, 20°06'16''.30		

STAR 43.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°39'47''.04	—0''.46	46''.58	20°19'06''.94	+0''.03	6''.97	
II	46 .71	— .46	.25	.77	+ .03	.80	
III	46 .88	— .45	.43	.72	+ .03	.75	
IV	47 .05	— .45	.60	.56	+ .03	.59	
V	46 .32	+ .22	.54	.68	— .02	.66	
VII	46 .27	+ .22	.49	.82	— .02	.80	
VIII	46 .32	+ .22	.54	.95	— .02	.93	
IX	46 .06	+ .23	.29	.74	— .02	.72	
Mean,			128°39'46''.46	Mean,			20°19'06''.78
Probable Error,			±.036	Probable Error,			±.035
Proper Motion,			—0.097 ±0.010	Proper Motion,			+0.007 ±0.010

STAR 44.

Plate.	Right Ascension :			Declination :			
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.	
I	128°40'18''.28	—0''.23	18''.05	20°36'14''.23	+0''.07	14''.30	
II	18 .27	— .23	18 .04	.33	+ .07	.40	
III	18 .49	— .22	18 .27	.31	+ .07	.38	
IV	18 .26	— .22	18 .04	.21	+ .07	.28	
V	18 .05	+ .11	18 .16	.40	— .03	.37	
VII	17 .95	+ .11	18 .06	.42	— .03	.39	
VIII	17 .87	+ .11	17 .98	.36	— .03	.33	
IX	18 .08	+ .11	18 .19	.33	— .04	.29	
Mean,			128°40'18''.10	Mean,			20°36'14''.34
Probable Error,			±.027	Probable Error,			±.013
Proper Motion,			—0.048 ±0.008	Proper Motion,			+0.015 ±0.004

STAR 45.

Plate.	Right Ascension :			Declination :		
	At Epoch of Plate.	P. M.	At Epoch 1875.0.	At Epoch of Plate.	P. M.	At Epoch 1875.0.
II	128°44'17''.78	—0''.33	17''.45	19°51'23''.45	+0''.08	23''.53
III	.62	— .33	.29	.38	+ .08	.46
IV	.98	— .33	.65	.22	+ .08	.30
V	.26	+ .16	.42	.44	— .04	.40
VII	.34	+ .16	.50	.56	— .04	.52
VIII	.39	+ .16	.55	.45	— .04	.41
IX	.23	+ .16	.39	.43	— .04	.39
Mean,			128°44'17''.46	Mean,		
Probable Error,			±.032	Probable Error,		
Proper Motion,			—0.070 ±0.009	Proper Motion,		
				+0.017 ±0.006		

The final results of the measurements have been collected on the next page; the right ascensions and declinations are obtained from α_1 and δ_1 , which are printed in the foregoing pages in slightly bolder type, by adding the transformation corrections given in Table VI, page 239. The magnitudes are those of the Bonn Durchmusterung.

Catalogue of the Relative Positions and Proper Motions of 42 Stars in the Praesepe Group.

Mean Equinox of 1875.0.

Epoch 1875.0.

Star.	Mag. B. D.	Right Ascension.	Proper Motion.	Declination.	Proper Motion.	No. of Plates.
1	8.8	127° 35' 27".78	—".091	+20° 26' 49".70	+".032	8
2	8.2	36 26.49	—.066	+20 09 59.43	+ .031	8
3	8.4	37 30.81	—.081	+19 57 07.85	+ .002	7
4	7.2	39 11.48	—.061	+19 42 04.48	+ .002	8
5	8.0	47 35.08	—.030	+20 38 32.69	+ .037	8
6	8.4	51 32.17	—.069	+20 25 32.80	+ .044	8
7	9.0	54 07.49	—.071	+19 56 14.22	— .006	8
7A	8.9	57 57.87		+20 09 42.23		4
8	9.0	57 58.28	—.072	+20 26 12.42	+ .025	8
10	8.0	58 48.92	—.040	+20 06 50.24	+ .029	8
11	8.8	127 59 37.19	—.075	+20 01 46.74	+ .021	8
14	8.0	128 07 44.78	—.042	+20 31 29.12	+ .027	8
15	7.0	07 54.88	—.054	+20 13 01.32	+ .025	8
16	8.0	08 44.32	—.073	+19 42 50.04	+ .006	8
17	7.2	10 04.92	—.058	+19 48 46.88	+ .018	8
18	8.2	11 31.77	—.046	+19 59 31.59	+ .021	8
19	9.0	11 36.50		+20 35 50.69		5
20	8.2	11 51.24	—.042	+19 58 50.44	+ .016	8
22	7.0	12 43.14	—.068	+20 26 50.10	+ .027	8
23	7.3	15 00.34	—.049	+20 34 39.01	+ .040	8
23A	9.0	15 56.81	—.069	+20 26 02.88	+ .020	8
24	8.2	16 57.45	—.045	+19 58 19.04	+ .015	8
25	8.5	17 31.22	—.054	+20 07 36.50	+ .015	8
26	7.0	17 35.17	—.063	+19 47 20.68	+ .007	8
27	7.3	17 52.03	—.035	+20 06 36.40	+ .032	8
28	8.5	18 06.49	—.040	+20 16 30.70	+ .015	8
29	8.8	18 42.24	—.062	+20 37 20.21	+ .036	8
31	7.2	* 19 10.91	—.057	+19 59 06.57	+ .022	8
32	9.0	20 53.84	—.059	+19 38 05.83	+ .007	8
33	8.2	22 26.97	—.072	+19 40 08.90	+ .009	7
34	7.1	23 07.75	—.058	+20 09 36.79	+ .022	8
35	8.7	25 05.96	—.058	+20 42 28.08	+ .030	8
36	9.0	26 25.91	—.095	+20 22 34.83	+ .033	7
37	7.7	26 30.02	—.059	+20 01 18.46	+ .011	8
38	9.0	29 56.77	—.045	+20 16 17.75	+ .016	6
39	8.6	30 39.81	—.075	+20 21 50.91	+ .015	8
40	8.7	32 14.58	—.090	+19 42 12.03	+ .010	8
41	9.5	36 07.23		+20 06 19.46		2
42	9.3	37 58.79		+20 06 13.76		2
43	7.5	39 47.65	—.097	+20 19 03.89	+ .007	8
44	8.9	40 22.88	—.048	+20 36 11.29	+ .015	8
45	8.4	128 44 12.34	—.070	+19 51 19.77	+ .017	7

VI.

Discussion of Results.

Let us first ascertain what is the probable error of a measured coördinate, being careful not to let personalities in the observing enter into our result. Each coördinate was measured completely, that is in both the direct and in the reversed positions, by two observers. The difference between the two complete measurements will be free from personalities and may be ascribed to errors of observation. This difference, which I shall call v , may easily be computed from Table III; say the two observers are Schlesinger and Kretz, then subtract ($S-K$) *direct*, from ($S-K$) *reversed* and the difference is double the amount by which one observer's complete measurement differs from the other's, or $2v$. The probable error of a final coördinate is then given by,

$$\pm \frac{0.6745}{2} \sqrt{\frac{[vv]}{n}}$$

Proceeding in this way for all the plates we obtain the following probable errors. Only those stars were used, thirty-three in number, which appear on all the plates.

		Probable Error of a final x .	Probable Error of a final y .
Plate	I	$\pm 0''.034$	$\pm 0''.031$
	II	.036	.029
	III	.023	.023
	IV	.024	.020
	V	.020	.027
	VII	.034	.020
	VIII	.032	.025
	IX	.037	.027
Means,		$\pm 0''.030$	$\pm 0''.025$

The greater uncertainty in right ascension is due to the fact that the images are usually elongated in that direction and are therefore more difficult to bisect. The elongation was caused by the failure of Rutherford's clock to keep pace exactly with the diurnal motion of the group, sometimes lagging slightly or sometimes moving too rapidly.

In the tabulation of results the probable error of each right ascension and declination is given. We may compute the probable error of right ascension of a star as derived from a single plate by the expression

$$\pm 0.6745 \sqrt{\frac{[vv]}{264 - 66}} \cos \delta_0$$

where $[vv]$ is the sum of the squares of all the residuals in right ascension for the thirty-three stars which appear on all eight plates; the factor $\cos \delta_0$ serves to reduce the probable error to arc of a great circle. The expression for the probable error of a single declination is identical with the above except that $\cos \delta_0$ is omitted. In this way we obtain the probable errors,

In Right Ascension.

$$\pm 0''.081$$

In Declination.

$$\pm 0''.058$$

If we do not confine ourselves to the thirty-three stars as above, but use all the stars, we get

$$\pm 0''.080$$

$$\pm 0''.060$$

Thus it appears that the uncertainty in a right ascension or in a declination is considerably greater than that in the corresponding measured coördinate. We may conclude from this that when a large number of plates is available, better results will be attained, for a given expenditure of time and labor, by measuring a large number of plates rather than measuring a few with all the elaboration used in the present research. But for the Rutherford photographs such elaboration is amply justified by the very limited number of existing photographs of so early a date.

It might appear at first as though a large part of the discrepancy between the two sets of probable errors, namely, those for the measured coördinates, and those for the resulting right ascension and declination, could be accounted for by the uncertainty of the constants used for the several plates. That such is not the case appears from the following considerations: the residuals for the five comparison stars, given on pages 247 and 248, exhibit a remarkable uniformity, showing that the greater part of these residuals is due to inaccuracies in the meridian observations. It follows, therefore, that the probable errors given for the constants p , r , k and c , are due not so much to errors in

measuring the plate as to errors in the meridian places. To obtain more precise information on this point, let us correct the meridian places of each of the comparison stars by the mean of the residuals for that star, and suppose we have effected the least-square solutions anew, using now the corrected meridian places. It can easily be shown that the new solutions would lead to exactly the same values of the constants as had been first obtained, but now each residual will be altered by a certain quantity, namely, the amount of the corresponding correction to the meridian place. We may then subtract at once the mean of the residuals for a star, from the corresponding residual in each least-square solution and then compute the probable errors of the constants. The results of such a computation are as follows:

	Probable Error of <i>p</i> or <i>r</i> .	Probable Error of <i>k</i> or <i>c</i>
Plate I	± 0.000013	$\pm 0''.026$
II	24	.049
III	16	.032
IV	16	.032
V	15	.030
VII	08	.016
VIII	11	.022
IX	20	.041
Means,	± 0.000015	$\pm 0''.031$

The former means were

$$\pm 0.000032 \qquad \pm 0''.065$$

and these must be regarded as indicating the uncertainty in the absolute values of the constants; if the constants which we have obtained are in error, then there will be a decided tendency to error in the same direction on *different* plates, and the smaller probable errors given above indicate how much we should expect the adopted values of the constants to differ from each other as obtained for different plates. Consequently only a small part of the discrepancy between the probable errors of the measured coordinates and of the right ascensions and declinations can be due to uncertainties in the adopted constants.

The discrepancy is probably caused by inaccuracies, and in some cases neglect, of instrumental corrections. For example, the difference between the two complete measurements of a coör-

dinate is independent of errors in the determination of the division corrections, because the two observers always used the same lines; but not so with the differences of the right ascensions or declinations as derived from different plates. Similarly, the corrections for temperature and straightness of the scale, which we have neglected, do not affect the agreement of the measured coördinates. Possibly too, there have been distortions of the film, but the smallness of the probable errors on the whole must rather be taken as evidence against such distortions. It is important to note that the close agreement of the right ascensions and declinations for different plates affords a striking confirmation of the permanence of the Rutherford plates, which in the present case have been measured a quarter of a century after they were made.*

If we consider the probable errors of the measured coördinates, we see that the uncertainty is considerably greater upon some plates than upon others. Notwithstanding, equal weights have been assigned to all the plates, since it appears that the uncertainty in a measured coördinate forms only a small part of the uncertainty in the corresponding right ascension or declination.

Let us now compare the photographic results with those of the heliometer. In his memoir upon the group, Professor Schur has given the places of forty-five stars referred to the mean equinox and epoch of 1875.0, which are the same as those used in the present paper. Of these stars all but five appear on the photographs. The following table gives first the uncorrected or direct differences obtained by subtracting the right ascension, declination and proper motion of each star in our catalogue, from the corresponding quantities in Schur's. The differences in right ascension and in proper motion in right ascension have been multiplied by $\cos \delta_0$ to reduce them to arc of a great circle.

* See, in this connection, "On the Permanence of the Rutherford Photographs," by Harold Jacoby, *Annals of the N. Y. Acad. of Sciences*, Vol. IX.

COMPARISON WITH HELIOMETER RESULTS.
 HELIOMETER *minus* PHOTOGRAPHS.

Star.	Right Ascension.		Declination.		Proper Motion.		No. of Plates.
	Direct Diff's.	Corr'd. Diff's.	Direct Diff's.	Corr'd. Diff's.	Diff's in R. As.	Diff's in Decl.	
1	—0.04	+0.07	+0.40	—0.08	+0.016	—0.015	8
2	— .03	— .02	+ .46	— .04	— .014	— .020	8
3	— .17	— .24	+ .61	+ .10	+ .011	+ .004	7
4	+ .24	+ .09	+ .73	+ .20	— .019	+ .005	8
5	— .15	+ .03	+ .67	+ .13	+ .021	— .001	8
6	— .05	+ .06	+ .55	— .02	— .011	— .032	8
7	+ .12	+ .06	+ .67	+ .07	— .004	+ .018	8
8	— .11	+ .01	+ .50	— .10	— .004	— .020	8
10	+ .11	+ .12	+ .72	+ .10	— .030	— .022	8
11	— .01	— .03	+ .67	+ .04	+ .003	— .013	8
14	— .26	— .10	+ .74	+ .08	— .032	— .008	8
15	— .11	— .06	+ .52	— .15	— .019	— .008	8
16	+ .38	+ .25	+ .57	— .12	— .007	+ .001	8
17	+ .22	+ .18	+ .70	+ .01	— .017	— .019	8
18	+ .16	+ .13	+ .62	— .08	— .023	— .028	8
19	— .27	— .09	+ .71	+ .04	— .035	— .022	5
20	+ .03	.00	+ .66	— .04	— .007	— .017	8
22	— .20	— .07	+ .67	— .02	— .020	— .031	8
23	— .08	+ .04	+ .79	+ .09	— .016	— .011	8
24	— .06	— .09	+ 1.04	+ .31	— .016	— .011	8
25	— .02	.00	+ .62	— .10	— .016	— .011	8
26	+ .20	+ .10	+ .69	— .05	— .014	— .006	8
27	+ .08	+ .10	+ .87	+ .14	— .032	— .033	8
28	— .25	— .18	+ .68	— .04	— .028	— .014	8
29	— .51	— .31	+ .73	+ .02	— .004	— .026	8
31	+ .01	— .02	+ .80	+ .06	— .007	— .017	8
32	+ .32	+ .17	+ .80	+ .04	— .014	— .002	8
33	+ .07	— .07	+ .74	— .02	+ .009	— .011	7
34	+ .07	+ .11	+ .94	+ .19	+ .002	— .010	8
35	— .32	— .09	+ .66	— .08	— .009	— .026	8
36	— .13	— .02	+ .79	+ .03	+ .037	— .030	7
37	— .06	— .07	+ .68	— .10	— .001	— .006	8
38	— .05	+ .03	+ .93	+ .14	— .018	— .008	6
39	— .14	— .03	+ .77	— .02	+ .008	— .010	8
40	+ .08	— .04	+ .57	— .25	+ .028	+ .001	8
41	— .11	— .09	+ .78	— .05			2
42	— .23	— .20	+ .66	— .18			2
43	— .16	— .06	+ .91	+ .07	+ .031	— .002	8
44	— .11	+ .09	+ .71	— .12	— .017	.000	8
45	— .07	— .13	+ .58	— .30	+ .018	+ .011	7

The corrected differences in right ascension and declination were obtained by adding systematic corrections and also by modifying the scale-value and orientation of the photographs. That is, a least-square solution was made to determine how much the

constants of the plates would have to be changed so as to secure the best possible agreement between the two catalogues. Each star gives two equations of the form,

$$\begin{aligned} Xdp + Ydr + dk + da &= 0 \\ Ydp - Xdr + dc + d\delta &= 0 \end{aligned}$$

where da and $d\delta$ are the uncorrected or direct differences in the table. The least-square solution may be carried out in a manner entirely similar to that previously used. The differences for stars 18, 20, 24 and 25 were not used because these stars were not included in Schur's triangulation, but each was merely located by position angle and distance from the nearest star in the triangulation. Stars 19, 41 and 42 were also excluded in making the least-square solution because of the small number of plates on which they appear. The remaining stars, thirty-three in number, give the following corrections to the contents:

$$\begin{aligned} dp &= +0.000011 \pm 0.000009 \\ dr &= +0.000098 \pm 0.000009 \\ dk &= +0''.047 \pm 0'',.014 \\ dc &= -0'',.667 \pm 0'',.014 \end{aligned}$$

The probable error of one equation is

$$\pm 0'',.080$$

a quantity which speaks well for the accuracy of all three researches concerned. The corrected differences in the table are now obtained by adding to each uncorrected difference

$$Xdp + Ydr + dk \text{ in the right ascensions,}$$

and

$$Ydp - Xdr + dc \text{ in the declinations.}$$

From the above value for dp we see that the meridian observations gave a scale-value which agrees very closely with that obtained from the heliometer places; the largest effect that dp has on either coördinate of any star is only about $0''.02$. On the other hand the value of dr , or the change in the orientation constant is quite large, corresponding to a correction of about $0''.20$ in the coördinates of outlying stars. The meridian observations which we used to determine the orientation of the group, were also employed by Schur for the same purpose, and were found by him to give results which practically agreed with those obtained by an independent method. As we have adopted Schur's proper motions for the comparison stars, to reduce their places to the epochs

of the plates, we can only conclude that the somewhat large value of dr is due to the fact that the relative positions of these stars with respect to the rest of the group have been differently determined by the photographs on the one hand, and the heliometers on the other. This explanation is borne out by the comparatively large values of the corrected differences for the comparison stars, numbers 4, 5, 15, 40 and 44.

The large value for dk , or the systematic correction in declination, was to be expected. We have already remarked (see page 236) that the proper motions used for the comparison stars were not derived by Schur from the direct differences between the two heliometer determinations of the places of these stars, but that systematic corrections,

$$+ 0^{\circ}.0003 \text{ and } - 0^{\circ}.039$$

were added to the proper motions in right ascension and declination respectively. Hence we must expect the photographic places to differ from those of the heliometer for the epoch of 1875.0, by

$$+ 0^{\circ}.071 \text{ and } - 0^{\circ}.612,$$

the proper motion having been used for an interval of 15.7 years. These corrections agree quite well with the values of dk and dc respectively, as obtained above.

VII.

Orientation by Trails. Scale-Value.

An independent method for orienting a stellar photograph is furnished by the "trails" or third images of some of the brighter stars. The Rutherford photographs previously reduced depend upon this mode of orientation, and the present research offers an admirable opportunity for testing its accuracy. Four trails have been measured and reduced on each Præsepe plate, and the resulting values of the orientation corrections were compared with the results obtained from a comparison with meridian observations, and also with those obtained with the use of the heliometer places. On Plate II the trails were too faint to admit of measurement, and on Plate V they were missing altogether.

The trails were measured in a different manner from that used for the other images. The plate was first set in the position which it occupied when " y direct" had been measured for the stars, and the micrometer was set and read on the east image of a star whose trail was to be measured. Then, without touching the microscope, the plate was moved along the cylinder till the corresponding trail came into view. This was always possible because the plate had been approximately oriented when first set in the machine. Two readings were made upon the trail and the plate was then moved back to the east image, which was read a second time. The same operations were gone through for the west image, and the mean of all the readings on the images was subtracted from the mean of the readings on the trail, thus giving the *offset* in declination by which the trail differed from the middle point between the two images. All the above operations were repeated in the opposite position of the plate, namely that corresponding to " y reversed," except that in the latter case the mean of the readings on the trail was subtracted from that for the images, so as to get the same sign for the offset as before. Each trail was thus measured by two observers separately, so that in all, sixteen readings were made on each trail, and eight upon each of the images. The resulting offsets are tabulated below in millimetres.

TRAIL MEASUREMENTS.

Star. Plate.	15.	22.	23.	27.	31.	37.
I	—0.0443		—0.0193		—0.0084	+0.0183
III	—0.0676		—0.0404	—0.0398	—0.0292	
IV	—0.0535		—0.0312		—0.0169	+0.0070
VII	—0.0772	—0.0422	—0.0499		—0.0294	
VIII	—0.1054	—0.0766	—0.0718		—0.0510	
IX	—0.0850		—0.0568		—0.0356	—0.0070

The distance from each trail to the middle point between the corresponding images was measured approximately as follows, being practically the same for all the stars upon a plate :

Plate, I III IV VII VIII IX

s, 35.0, 35.1, 35.0, 39.3, 48.1, 39.4 millimetres.

We shall now consider what corrections must be applied to the above offsets in order that the true orientations of the plates may be computed from them.

Instrumental Corrections. The only correction of this kind is that for rotation, the data for which have already been given in Table II. Using the same notation as before the correction to the offset is

$$-s \cdot i \cdot \sin 1''$$

which is the same for all four stars. Having applied this connection, the offset may now be converted into seconds of arc by multiplying by the approximate scale value, 52.87.

Transformation Corrections. For the present purpose it will be convenient to use Ball and Rambaut's formulas quoted on page 238, in which $X \sec \delta_0$ and Y appear in the second members instead of $\Delta\alpha$ and $\Delta\delta$. We need only the second of these formulas :

$$\Delta\delta - y = -\frac{1}{4} (X \sec \delta_0)^2 \sin 2\delta_0 - \frac{1}{8} Y^2 - (X \sec \delta_0)^2 Y$$

For the trail, $X \sec \delta_0$ is diminished by

$$z = 52.87 \cdot s \cdot \sec \delta_0,$$

while Y remains practically unchanged. Hence the correction to the offset is,

$$+ \frac{1}{4} \sin 2\delta_0 \cdot z \cdot (z - 2X \sec \delta_0) + z \cdot (z - 2X \sec \delta_0) Y$$

Refraction Corrections. The trails were taken somewhat later than the principal images of the group, and as the zenith distance

changed in the interval, the refraction-coefficients will also be changed. Denoting by M_y' and N_y' what these coefficients become for the trails, we have the correction to the offset,

$$M' \cdot z + (M_y - M_y') X \sec \delta_0 + (N_y - N_y') Y$$

The first term is constant for all four stars, and the two remaining terms are small. To calculate M_y' and N_y' we must know how much the hour angle has changed in the interval between the exposures for the principal images and that for the trail. As each of the former lasted six minutes and as the exposure for the trail was much shorter, we may safely adopt seven minutes of time as the change in the hour-angle. M_y' and N_y' may then be calculated with sufficient accuracy by interpolating in Table VII.

After these corrections have been applied it will be convenient to transform the offsets into position angles, which may be done by the formula

$$p = 270^\circ + \sin^{-1} \left\{ z \cdot \frac{\text{offset}}{\cos \delta_0} \right\}$$

Precession, Nutation and Aberration. Formulas for correcting position angles for these were deduced in convenient form by Bessel*; let

$$a' = 20'' \sec \delta_0 \sin a_0$$

$$\beta' = \sec \delta_0 \cos a_0$$

$$\gamma' = \tan \delta \cos a_0$$

$$\delta' = \tan \delta_0 \sin a_0$$

A, B, C, D = Bessel's star-numbers, tabulated for each day in the year in the ephemerides.

The true position angle at the beginning of the *same* year is found by adding to the observed position angle the correction,

$$(-Aa' + B\beta' + C\gamma' + D\delta')$$

Then to reduce this to beginning of *another* year we add

$$+ 20''.06 \sec \delta_0 \sin a_0 \cdot t$$

where t is the integer corresponding to the difference of the years, and must be considered positive if we are reducing an observation to a later year than that in which it was made.

As an example of the reduction of trail measurements, I have set down the calculations in detail for the trail of Star 23, Plate I.

* "Astronomische Untersuchungen" Vol. I., pg. 202.

Offset,	— 0.0193	millimetres
Rotation Corr'n.,	+ 0.0005	
	— 0.0188	
In arc,	— 0''.99	
Transf. Corr'n.,	+ 1 .75	
Refraction	+ 0 .24	
Corrected offset	+ 1''.00	
Position Angle,	270 + 111''.9	
— ($A\alpha' + B\beta' + C\gamma' + D\delta'$),	+ 2 .6	
20.06'' sec $\delta_0 \sin a_0 \cdot t$,	+ 84 .0	
True Position Angle,	270° + 198''.5	

Consequently we have from this star,

$$r = + 0.000963$$

Similar calculations for all the trails gave the following results in which r has been multiplied by 10^6 throughout.

ORIENTATION BY TRAILS. VALUES OF $r \times 10^6$.

Star. Plate.	15.	22.	23.	27.	31.	37.
I	+954		+963		+840	+873
III	+334		+390	+118	+288	
IV	+851		+777		+757	+706
VII	—128	+186	—140		— 52	
VIII	+122	+152	+111		+113	
IX	—360		—341		—248	—254

Taking the mean for the four stars on each plate and setting down again the values of r previously obtained by comparison with the meridian observations, we have,

	Orientation by Trails.	Orientation by Merid. Obs.
Plate I	+ 0.000908	+ 0.001054
III	+ 282	+ 409
IV	+ 773	+ 1435
VII	— 34	— 35
VIII	+ 124	+ 211
IX	— 301	— 326
Means	+ 292	+ 458

In comparing these it will be remembered that a difference of 0.000100 corresponds to about 0''.20 in the coördinates of the outlying stars of the group. The results are decidedly adverse to

the accuracy of this mode of orientation, especially as a comparison with the heliometer places indicates a further correction of

$$+ 0.000098$$

to the orientations obtained by using the meridian places. The large discrepancies are probably due to jarring of the plate during exposure, caused by stopping and starting the clock-work several times; the large difference for Plate IV admits of no other obvious explanation.

Let us now examine the scale-values of the different plates. The values of p given at the beginning of Section V include aberration and temperature effects. Formulas for the former correction are thus given by Bessel:*

$$\begin{aligned}\gamma &= -(\cos \delta_0 \sin a_0 + \tan \omega \sin \delta_0) \\ \delta &= +(\cos \delta_0 \cos a_0)\end{aligned}$$

Then the true distance is found by adding to the observed distance s ,

$$-s(C\gamma + D\delta),$$

C and D being as before, the Besselian star numbers.

We may also correct the values of p for the temperature at which the plates were measured by adding

$$+ 0.0000017 (T - 65^\circ),$$

T being the temperature in Fahrenheit degrees at which the plate was measured, given in Table II. This expression is easily derived from the value of v on page 223. Corrections for the temperature at which the plate was exposed ought also to be applied, but sufficient data to establish a connection between this quantity and the scale-value are lacking. After a greater number of Rutherford's photographs have been reduced we may have more definite information on this point.

The true scale-value S (so far as it can be obtained without the last correction), is given thus,

$$S = 52''.87 [1 + p - C\gamma - D\delta + 0.0000017 (T^\circ - 65^\circ)]$$

The following table gives the corrections and the resulting scale value for each plate. The corrections for temperature are very small and might well have been neglected. The last two columns give the readings of the thermometer attached to the telescope and of the "focus," which have been copied from Table I for convenience of reference.

* "Astronomische Untersuchungen," Vol. I, page 208.

SCALE-VALUE.

Plate.	Cor. for Aberration.	Cor. for Temp.	Corrected Scale-Value.	Tel. Therm.	Focus.
I	—0.000099	0.000000	52.8701	+58°	8.4
II	— 99	+	52.8715	58	8.4
III	— 100	— 3	52.8712	53	8.4
IV	— 100	+	52.8760	53	8.4
V	— 98	+	52.8788	48	7.8
VII	— 99	— 2	52.8827	58	7.7
VIII	— 99	— 5	52.8831	58	7.7
IX	— 98	— 2	52.8840	48	7.8

The mean of the scale-values is

$$52''.8772$$

and if we adopt the correction of + 0.000009 as indicated by comparison with the heliometer places, this becomes

$$52''.8776.$$

However, either of these must still be regarded as only an approximate value, since the separate values for the different plates, as given above, vary in a way that cannot be fully explained by a connection with the readings either of the telescope thermometer or of the "focus."

The above investigations on the orientation and on the scale-value lead to the same conclusion; it will usually be better to determine all the constants of a plate by comparing the measures of some of the stars with their positions as known through meridian observations or otherwise, than to attempt to reduce them by means of a predetermined scale-value and orientation. In any case it is necessary to appeal to such known positions to determine the values of k and c , or the absolute place of the group in the sky. The positions of two stars are theoretically sufficient to determine all four constants, but in most cases it will be possible to find enough stars to eliminate errors of observation to a large extent.

In conclusion, I wish to acknowledge my indebtedness to Messrs. Kretz and Hays for assisting me in the measurement of the plates, and to Professor Jacoby, who has kindly explained to me the methods used by him in the measurement and reduction of stellar photographs, and who has also suggested some improve-

ments in the paper in reading over the proofs. Finally I desire to express my thanks to Professor Rees, Director of the Observatory, for the interest he has shown in my work, and for securing its publication.

Note on Refraction Formulas for Photographic Plates.

Formulas for correcting the measured rectangular coördinates of a star upon a photographic plate for refraction, may be easily derived from the well known general formulas of Bessel. On page 166, Vol. 1. of his "Astronomische Untersuchungen" he gives the following corrections to the differences of right ascension and declination:

$$\begin{aligned}\Delta (a' - a) &= s \cdot k [\tan^2 \zeta \cos (p - q) \sin q - \tan \zeta \sin q \tan \delta_0 \cos p \\ &\quad + \sin p] \sec \delta_0 \\ \Delta (\delta' - \delta) &= s \cdot k [\tan^2 \zeta \cos (p - q) \cos q + \tan \zeta \sin q \tan \delta_0 \sin p \\ &\quad + \cos p]\end{aligned}$$

Substituting

$$\begin{aligned}X &= s \sin p \\ Y &= s \cos p \\ G &= \tan \zeta \sin q \\ H &= \tan \zeta \cos q\end{aligned}$$

we obtain

$$\begin{aligned}\Delta (a' - a) &= k X \sec \delta_0 (I + H^2) + k Y (G - \tan \delta_0) H \sec \delta_0 \\ \Delta (\delta' - \delta) &= k X (G + \tan \delta_0) H + k Y (I + G^2)\end{aligned}$$

These formulas become identical with those of Professor Jacoby when we change k into β in order to allow for the increased refrangibility of photographic rays.

One point in the above deduction deserves mention; the quantities δ_0 , etc., were intended by Bessel to be the means of corresponding quantities for the two stars whose distance along the arc of the great circle joining them has been measured. We have treated them as though they referred to one end of that arc; however, this merely amounts to neglecting terms in the second and higher powers of s , which may be done for most photographic plates.

If we omit the middle term in each bracket in Bessel's formulas we obtain the formulas given by Professor Turner; the omission of these terms, as has been repeatedly pointed out, corre-

sponds to a rotation of the axes, and is, therefore, of no importance when we determine the constants of a plate by comparing the measured coördinates of some of the stars with their known places. Turner's formulas are

$$\begin{aligned}\Delta X &= kX \cdot (1 + H^2) + kY \cdot GH \\ \Delta Y &= kX \cdot GH + kY \cdot (1 + G^2)\end{aligned}$$

These formulas may be simplified when we use the above method for determining the constants, as I pointed out in the *Astronomical Journal*, No. 430; rejecting so much of the correction for refraction as may be regarded as either an orientation correction or a scale-value correction, we have remaining

$$\begin{aligned}\Delta X &= kX \cdot (H^2 - G^2) \\ \Delta Y &= kX \cdot 2GH\end{aligned}$$

These formulas might have been used for the reduction of the Præsepe plates, but as we wished to know the true orientation and scale-value for each plate, extra corrections to these constants would have been necessary. Only four of the comparison stars used need corrections for refraction; so that in the present case nothing would have been gained by the use of the last formulas. When, however, the number of comparison stars is greater, or when we do not care especially to know the true orientation and scale-value of a plate, these formulas will save some labor.

"There shall be appended to each dissertation a statement of the educational institutions that the author has attended, a list of the degrees and honors conferred upon him, as well as the titles of any previous publications." *Extract from Section 8.—Regulations for the Degree of Doctor of Philosophy, established by the University Council.*

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